Reassessing socioeconomic inequalities in mortality via distributional similarities

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Abstract

Commonly used measures of socioeconomic inequalities in mortality, such as the slope index and the relative index of inequality, are based on summary measures of the group-specific age-at-death distributions (e.g. standardized mortality rate or life expectancy). While this approach is informative, it ignores valuable information contained in the different distributions. We propose a mortality inequality measure that readily captures the distributional difference between two or more population's subgroups. Leveraging a metric of statistical distance, our Population Total Variation (PTV) measure is sensitive not only to changes in the means or variances, but also to broader mortality changes that affect distributional shapes. We use observed mortality data by socioeconomic groups to asses mortality inequalities with both established measures and our proposed PTV. Our findings suggest that conventional summary-based measures can bias our understanding of socioeconomic inequalities in mortality. We present applications based on educational groups and groups defined by an area-level deprivation measure to exemplify how the PTV can be applied in different data availability contexts. We conclude that measuring distributional similarities in mortality enhances our understanding of between-group inequalities in mortality.

What is already known

- Assessment of socioeconomic inequalities in mortality and their changes over time may depend on the measure employed.
- Typical approaches to measure such inequalities are based on summary measures of the population's subgroups mortality experience, disregarding either valuable information contained in their age-at-death-distribution, or the mortality experience of some groups, or both.
- One recent study showed that comparing distributional differences in mortality by income quintiles in Finland revealed different patterns of mortality inequality than those derived from commonly used measures.

What does this paper add

- A novel measure that readily captures distributional difference between two or more population's subgroups.
- Using distributional differences to quantify socioeconomic inequalities in mortality can alter our assessment of levels and trends of such inequalities across populations.
- We measure socio-economic inequalities in mortality in two applications with different data requirements: educational groups and groups defined by an area-level deprivation measure.

1 Introduction

A long-standing literature has shown that patterns of mortality can drastically differ between groups in a population and across geographical areas inside a country, producing socioeconomic inequalities in mortality (Illsley and Le Grand, 1993; Kunst and Mackenbach, 1994; Mackenbach et al., 2019). Evidence suggests that socioeconomic disadvantage often results in health disadvantage, leading to a social gradient in mortality. Low educated groups or those living in deprived areas tend to have lower life expectancy (Mackenbach et al., 2016; Murtin et al., 2022; Seaman et al., 2019a; Toch-Marquardt et al., 2014). This finding has sparked interest in measuring differences in mortality between groups within a population or country.

Differences across groups are generally assessed by comparing summary measures of mortality for each group, such as age-standardized mortality rates, life expectancy, modal age at death and, more recently, lifespan variation measures (Harper and Lynch, 2016). These measures are convenient as they are easily interpretable and because they summarize each group's mortality information into a single number. However, this approach can hide important differences in the underlying mortality patterns, and therefore provide an incomplete or biased assessment of socioeconomic inequalities in mortality. For example, one recent study showed that comparing distributional differences in mortality of groups defined by income quintiles in Finland revealed different patterns of mortality inequality than those derived from commonly used measures (Shi et al., 2023). Assessing socioeconomic inequalities in mortality using the full age-at-death distribution seems theoretically preferable, as it allows to consider all dimensions of inequality simultaneously rather than a single dimension only (e.g. mean or variance) (Edwards and Tuljapurkar, 2005; Sasson, 2016).

To illustrate this, we present four hypothetical populations in Figure 1 along with their respective level of socioeconomic inequality in mortality derived from different measures (see Section 2.1 for more details on these measures). The top panels of the figure show two subpopulations in two points in time or from two different countries. The range or difference in life expectancy between the subgroups, a common measure of mortality inequality, is the same in both panels and equal to 18.8 years, suggesting constant inequality. Furthermore, the range in lifespan variation, measured with the standard deviation of the ages-at-death, increases from 0.7 to 2.2 between panels A and B, which suggests worsening inequality. However, the overlap of the two distributions is greater in panel B than in panel A, indicating that more people share similar lifespans, or equivalently, that there is more equality between the two subgroups. Moreover, when using more than two subpopulations, the range or ratio, are restricted to comparisons between the two extreme distributions. With such approach, no distinction would be made between panels C and D, as only the intermediate group is different. However, it is evident from panel D that two subgroups have almost identical age-at-death distributions, suggesting greater equality than in panel C. Even using the slope index of inequality (SII), designed to incorporate information on the intermediate groups, the change from panel C to D would not be reflected in the measure as it remains constant under both scenarios[1]. These conclusions also hold in the analysis of relative measures of inequality (see Figure S1 in the Supplementary Materials).

These examples suggest that patterns of convergence/divergence between groups may be hidden when the measurement of socioeconomic inequality in mortality is restricted to the comparison of summary measures of mortality. Furthermore, given the regular shape of human mortality, there is a large overlap between different socioeconomic groups' age-at-death distributions, which have more similarities than dissimilarities. In other words, many individuals from different groups can have similar lifespans (Shi et al., 2023; Vaupel et al., 2021). Consequently, relying solely on age-standardized mortality rates, life expectancy, lifespan variation, or a combina-

[1]Notice that the SII remains constant only in the particular case of three equal sized subpopulations, this behaviour does not extend to other cases.

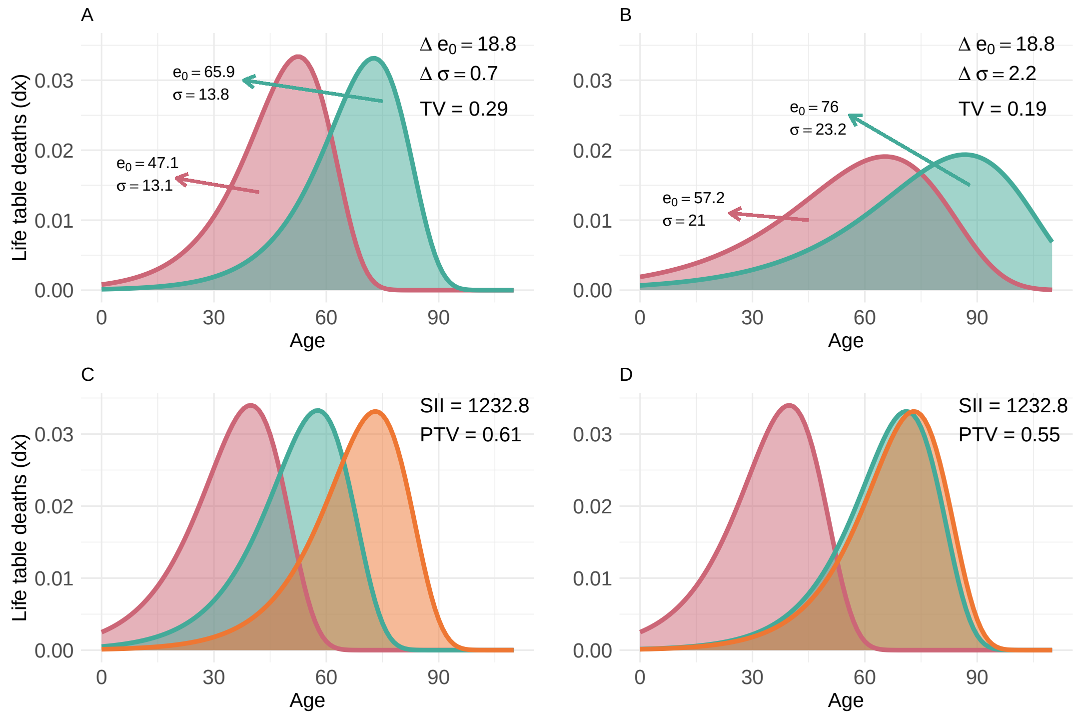

Figure 1: Hypothetical scenarios of the age-at-death distributions of two (Panels A and B) or three (Panels C and D) population's subgroups.
Note: The annotations on the graphs include: life expectancy at birth (e_0), lifespan variation (measured by the standard deviation of the ages-at-death, σ), difference (range) in life expectancy at birth between both distributions (Δe_0), difference (range) in standard deviation of the ages-at-death between both distributions ($\Delta\sigma$), total variation distance (TV), slope index of inequality for age-standarized mortality rates (SII) and population total variation (PTV).
Source: Authors' own elaborations.

tion of them may be insufficient to evaluate socioeconomic inequalities in mortality.

As such, our aim is to evaluate whether measuring the distributional difference between age-at-death distributions provides new and additional insights on socioeconomic inequalities in mortality, beyond what we can already derive from summary mortality measures. For this purpose, we introduce the Population Total Variation, a multivariate extension of a well-known statistical measure widely employed in other fields.

2 Background

2.1 Measures of socioeconomic inequality in mortality

Different measures to evaluate socioeconomic inequalities in health and mortality have been developed in the literature. These measures vary in complexity, in their incorporation of different population groups, and more importantly, in conceptual issues and implicit assumptions about inequality. Several studies have analyzed the strengths and limitations of different measures of health/mortality inequality

(Harper and Lynch, 2016; Mackenbach and Kunst, 1997; Regidor, 2004a,b). One of the choices to be made concerns which type of inequality one seeks to measure: inequality between different groups, inequality within a group, or both (Asada, 2013). In this paper, we focus on between-group inequality while considering within-group variation.

Table 1 presents the most commonly used approaches to measure socioeconomic inequality in mortality in the field of demography, along with their descriptions and properties. Perhaps the simplest methods are the range and ratio measures, whereby inequality is computed by looking only at the most and least advantaged groups, disregarding any information from other groups. To overcome this issue, measures that account for all subgroups, such as the slope and relative index of inequality (SII and RII), were developed (Mackenbach and Kunst, 1997; Pamuk, 1985; Preston et al., 1981). These measures quantify the social gradient in mortality by a weighted regression between the subgroups' mortality measures (generally the age-standardized mortality rate) and their relative rank in terms of socioeconomic status. Other important measures used in the broader field of socio-economic inequalities in mortality are the population attributable risk (PAR) and the population attributable fraction (PAF) (Harper and Lynch, 2016).

These commonly used measures of socioeconomic inequality in mortality are based on group averages (e.g. age-standardized mortality rate or life expectancy) or inter-group variation measures (lifespan variation), disregarding additional information contained in the age-pattern of mortality, as shown in Figure 1. Previous studies have applied them to estimate the range of life expectancy for the high and the low-educated groups (Zazueta-Borboa et al., 2023), range and ratio of the age-standardized mortality rates (Murtin et al., 2022), the slope and the relative indexes of inequality of lifespan variation (Seaman et al., 2019b), or the difference between mortality indicators (life expectancy and lifespan variation) of each group and those of the overall population (Trias-Llimós et al., 2023), among others. All of these studies rely on the comparison of summary measures from the first two moments of the distributions, ignoring additional and valuable information contained in the different distributions. Furthermore, studying these two moments separately may be confusing, as the the socioeconomic gradient between both indicators may move in different directions.

In the recent study of mortality inequalities, there is a consensus on the need to go beyond the mean and incorporate information on other aspects of the distribution of health that shed light on the heterogeneities that are usually masked when looking at average measures (Asada, 2013; Permanyer et al., 2023). In the field of demography, the most common approach to overcome this limitation is to study lifespan variation and its determinants along with life expectancy (van Raalte et al., 2018).

Lifespan variation has been incorporated in the study of socioeconomic inequal-

Table 1: Common measures of socioeconomic inequalities in mortality

Measure	**Formula and description**	**Properties**
Range/ Ratio	$$\text{Range} = m_k - m_1$$ $$\text{Ratio} = \frac{m_k}{m_1}$$ The difference/ratio of the measure of mortality between the most advantaged and the least advantaged group.	It is a simple and readily interpretable measure, and it can be applied to non-ordinal socioeconomic variables. However, it only reflects information of the extreme groups.
Slope index of inequality	$$m_i = \alpha + \beta R_i$$ $$\text{SII} = \hat{\beta}$$ It is the slope coefficient ($\hat{\beta}$) of the regression line between the group-specific mortality measure against their relative rank of socioeconomic status.	It measures the socioeconomic gradient in the mortality measure. It reflects the patterns of all social groups and considers the proportion of population in each group. It is often estimated by weighted least square regression, though other models have been proposed. However, it can only be applied to ordered groups.
Relative index of inequality	$$m_i = \alpha + \beta R_i$$ $$\text{RII} = \frac{\hat{\beta} + \hat{\alpha}}{\hat{\alpha}}$$ It is the relative counterpart of the SII. It can also be estimated as $\hat{\beta}/\bar{m}$.	

Notes: Let m_i be the mortality measure (life expectancy, lifespan disparity, median age at death, etc.), w_i the population share, $R_i = \frac{1}{2}w_i + \sum_{j=1}^{i-1} w_j$ the relative rank of group i, where $i \in 1, .., k$, and k is the number of groups, and $\bar{m}$ the mean of the mortality measure of all groups.

ities in mortality in two ways. On the one hand, it has been used to complement the comparison of group-specific life expectancy, by including the comparison between group-specific lifespan variation (van Raalte et al., 2018). On the other hand, considerable efforts have been devoted to measure the influence that population partitions have on the overall level of lifespan variation. For this purpose, several decomposition methods have been proposed to disentangle the contribution of between-group variance (between-group component) and individual stochasticity (within-group component) to the overall level of lifespan variation (Permanyer et al., 2023, 2018; Seaman et al., 2019a). Empirical studies have found small contributions of the between-group variance to the overall lifespan variation (Seaman et al., 2019a; van Raalte et al., 2012), which suggests that subgroups may be more similar than what is reflected by the comparison of life expectancy and lifespan variation separately (Shi et al., 2023). This finding has prompted further research to disentangle between-group differences.

In one of the most recently proposed decomposition methods, Permanyer et al. (2023) combine the individual and the group perspectives to give an overall picture of inequality in mortality while maintaining information on group-based inequalities. Their approach provides information about both, the distance between groups and their corresponding relative positions – the average advantage of one group in relation to another group. This approach is closely related with recent approaches in demography that are based on the age-at-death distributions.

2.2 How has the age-at-death distribution been used before?

Over time demographers have recognized the value of the information contained in age-at-death distributions and have used it to answer different research questions related to mortality inequalities. Is mortality converging across countries or across socioeconomic groups (Edwards and Tuljapurkar, 2005; Sasson, 2016)? What is the probability that an individual in one population outlives an individual in another population (Vaupel et al., 2021)? What is the degree of stratification of lifespans by social characteristics (Shi et al., 2023)? We recover the arguments put forward by all these studies to answer the question: Can the measurement of socioeconomic inequalities in mortality be refined by using the whole information of the age-at-death distribution?

Statistical distance or divergence measures can be employed to estimate the distance or similarity between two age-at-death distributions. Some previously used measures are the Shannon entropy (Bergeron-Boucher et al., 2020), the Tanimoto index (Shi et al., 2023) and Kullback-Leibler divergence (KLD), the latter being the most frequently used (d'Albis et al., 2014; Edwards and Tuljapurkar, 2005; Sasson, 2016).

The KLD or relative entropy is a measure of distributional divergence frequently used in the field of information theory. It quantifies the amount of information that would be lost if one distribution is used to estimate another. In demography, it has been used to evaluate mortality convergence across countries (d'Albis et al., 2014; Edwards and Tuljapurkar, 2005) and between education groups (Sasson, 2016). Under near-normality assumptions, the KLD can be decomposed into two parts: one reflecting differences in means and one reflecting differences in variances (Roberts and Penny, 2002). Evidence from this decomposition varies according to the groups analysed. When looking at mortality convergence across countries, groups vary mostly because of differences in standard deviation of the ages at death (Edwards and Tuljapurkar, 2005). For education groups, groups vary mainly due to differences in means (Sasson, 2016). The KLD is asymmetric (meaning that the KLD from distribution A to B is typically different from the KLD from B to A), consequently researchers need to define a reference population, with common choices being the period average distribution or the population with highest life expectancy or lowest lifespan variation.

Recent focus has been given to the fact that the comparison of life expectancy conceals similarities in the age patterns across groups. That is, the fact that the life expectancy of a group (X) is higher than that of another (Y) does not imply that all individuals from group Y will die before all individuals from group X, but rather that on average they will die sooner. At the individual level, the out-survival probability performs all possible pairwise comparisons to estimate the probability that a random individual from group X outlives a random individual from group Y. In this way, it explicitly takes into account the experience of each member of the population (Permanyer et al., 2023; Vaupel et al., 2021).

At the group level, the Tanimoto index has been employed to measure social stratification across groups (Shi et al., 2023). Specifically, the authors use the complement of the Tanimoto index to quantify the proportion of non-overlapping area between two age-at-death distributions. Their results have shown that comparing distributional differences in mortality revealed different patterns of mortality inequality than those derived from life expectancy and lifespan variation.

Although these three lines of research leverage age-at-death distributions, which is a change of paradigm compared to the usual measures, we find two limitations with these methods. First, when studying more than two groups, these methods rely on pairwise comparisons of the groups' distributions (which becomes computationally intensive for several groups), failing to summarize inequality into a single number or having to average all pairwise comparisons. Secondly, the KLD and the out-survival probability rely on the subjective choice of a reference distribution. Conceptually, if one were to extend the out-survival probability for more than two distributions, one needs to choose a group for which to measure its out-survival chances compared to the other groups, which can be seen as choosing a reference distribution. Out of the three approaches, the one by Shi et al. (2023) is the only one which does not require a reference distribution as it focuses on differences between distributions.

The concept of overlap is new in demography but not in other fields. In ecology, it has been used to study niche overlap (Mason et al., 2011). It has also been applied to evaluate targeting methodologies of social programs (El-Sheneity and Gadallah, 2017) and in applied psychology (Pastore and Calcagnì, 2019), among other applications.

In the following section we present the population total variation, our chosen statistical distance to measure inequalities in mortality between more than two groups. Our approach is similar to that of Shi et al. (2023); however, we extend their proposal by providing a straightforward approach to measure inequality between more than two groups, which does not necessitate performing all pairwise comparisons of the different groups. In addition, we apply our measure to different data availability contexts, including cases where individual-level data to partition the population is not readily available.

3 Methods

3.1 Population total variation

Probability metrics quantify the distance between two statistical objects, such as random variables or samples. For our purposes, we focus on probability distributions, specifically age-at-death distributions derived from a life table. Some examples of probability metrics are the total variation, Kullback-Leibler divergence, Hellinger distance, and χ^2 distance, among others.

The total variation is a measure of distance between two probability distributions that has been widely used in different fields. It has a natural interpretation: it is the non-overlapping area between the curves of two densities divided by the possible maximum non-overlapping area (equal to two) (Gibbs and Su, 2002). In our context, let d_x^i denote the life table age-at-death distribution of group i at age x, and let α and ω represent the first and last ages in the life table. The age-at-death distribution is a proper density, meaning that $\sum_{x=\alpha}^{\omega} d_x^i = 1$. The total variation at age α is calculated as follows: $\text{TV}_\alpha = \frac{1}{2}\sum_{x=\alpha}^{\omega} |d_x^1 - d_x^2|$, where d_x^1 and d_x^2 are the age-at-death distributions for the two populations. Up to our knowledge, the total variation has not been previously employed in demographic studies.

Using the same framework, we extend the total variation for more than two groups to represent the area outside the overlapping area between all densities divided by the possible maximum non-overlapping area. The maximum non-overlapping area would occur when the curves never overlap, and it would be equal to the number of groups. Formally, the population total variation at age α for n population's

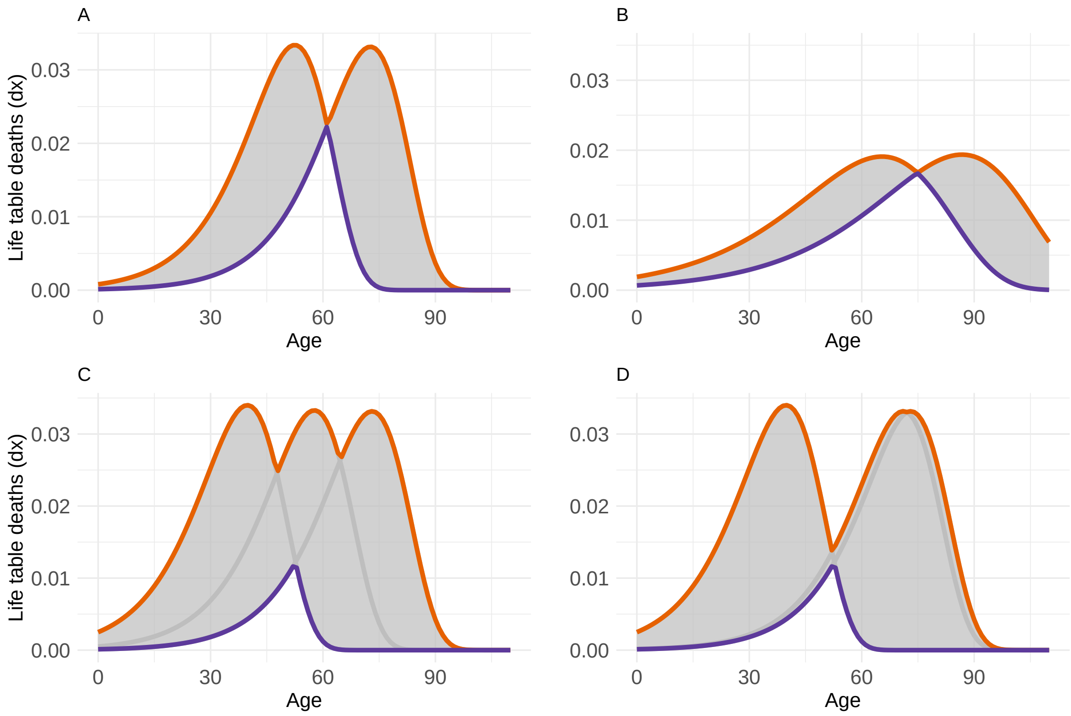

Figure 2: Areas considered in the computation of the TV (upper graphs) and PTV (lower graphs), which is the area contained between the maximum (orange) and the minimum (purple) d_x^i at each age.
Source: Authors' own elaborations.

subgroups is defined as:

$$\text{PTV}_\alpha = \frac{1}{n}\sum_{x=\alpha}^{\omega}\left[\max\{d_x^1, d_x^2, ..., d_x^n\} - \min\{d_x^1, d_x^2, ..., d_x^n\}\right] \tag{1}$$

where d_x^i is the life table age-at-death distribution of group i at age x. We call this measure population total variation (PTV) to differentiate it from the total variation (TV), which refers to two groups only. However, in the special case of only two groups, the two measures are identical (i.e. PTV=TV).

For a graphical representation of the two areas considered in the computation of the TV and PTV, Figure 2 shows such areas for the distributions presented in Figure 1.

3.2 Properties

The total variation distance has some useful properties for the comparison of age-at-death distributions. Given that the PTV is an extension, it inherits some of its properties. It is symmetrical, meaning that the result is independent of the order of the groups under comparison. This implies that the PTV can be applied to measure between group mortality inequalities with any population grouping (race, education, area-level indicators, occupation), and not just to ordinal categories such as income level. The PTV ranges between 0 and 1. Zero denotes absence of inequality and it is attained if and only if all age-at-death distributions are exactly the same. On the contrary, the maximum value of one is reached when none of the distributions overlap. This is a highly unlikely scenario for human mortality given the restricted domain of the age at death and the regular pattern of mortality. Both limits are unique, and can only be attained in the conditions mentioned before.

The PTV is sensitive to changes in the distribution of all subgroups, not only the extremes. The lower row of Figure 1 shows two scenarios in which the only difference is the middle distribution. In Panel C, the PTV is 0.61, while in Panel D it is 0.55. It is important to notice that if the middle group were to move towards the lower group instead of the higher one (as shown in panel D), the total variation would also decrease compared to panel C.

One key difference between the PTV and the KLD is that the former does not require a reference distribution nor distributional assumptions, while the latter does. For the PTV, inequality between sub groups diminishes when the distributions converge, regardless of where the convergence occurs and the shape of the underlying distributions. Consequently, the PTV does not assume that all groups should move towards a *best practice* shape with high life expectancy and low lifespan variation. As such, it is possible for two populations with significantly dissimilar internal distributions by group to share the same PTV.

It is important to notice that the PTV remains constant under certain transformations in the distributions. Specifically, if all distributions shift horizontally by the same magnitude, then the PTV will not change. This particular insensitivity is a desirable property as changing mortality does not necessarily mean changes in between group differences.

Finally, it should be noted that in the case of two populations, the total variation distance is closely related to the non-overlap index that has been recently used by Shi et al. (2023). For two distributions, the non-overlap index is given by the non-overlapping area between two curves (i.e. the total variation measure) divided by the total area under both curves, counting the overlapping area only once. A proof for the general case of this relation can be found in Stine and Heyse (2001).

4 Results

In this section, we use available life tables by socioeconomic status to provide some empirical analysis of socioeconomic inequalities in mortality by comparing our proposed PTV measure with other conventional indices: the range of life expectancy, the range of lifespan variation (measured with the standard deviation of the ages-at-death), and the slope index of inequality (SII) of age-standardized mortality rates. Comparisons with relative indices of inequality are presented in the Supplementary Materials. We use the standard deviation of the age-at-death distribution to measure lifespan variation due to its simplicity for interpretation and the fact that it is in the same scale as life expectancy (years). Moreover, measures of lifespan variation are highly correlated with each other (Van Raalte and Caswell, 2013; Wilmoth and Horiuchi, 1999), so the choice of measure does not affect our main conclusions to a great extent.

We start by analysing mortality inequality by educational groups in Denmark and Sweden from 1991-1995 to 2011-2015 in Subsection 4.1. We then move to the study of socioeconomic inequality by area-level deprivation index in England from 2006-2008 to 2014-2016 in Subsection 4.2. All of our analyses and results are fully reproducible using the data and codes provided in the open-access repository available at [link].

4.1 Mortality inequality by education in Denmark and Sweden

Information on the socioeconomic status of older individuals is limited, even in Nordic countries with high quality register data such as Sweden and Denmark. This limitation is often worked around by restricting the age range in the analysis. Németh et al. (2021) developed a non-parametric approach to reconstruct the education-specific composition and mortality curves of the older population. The method redistributes cases with unknown educational attainment and extrapolates the mortality curves and population shares by education level from the last available age-group with complete information on education. Using the estimated mortality values, they construct life tables by sex and education level (see Németh et al. (2021) for details on the methodology).

We use the life tables by education estimated by Németh et al. (2021) for Denmark and Sweden. Abridged life tables, starting from age 30, were estimated by sex for 5-year age groups and 5-year periods from 1991-1995 until 2011-2015 for three education levels. The open-age group is 90 years and above. The Swedish data only covers the population born in Sweden. Education categories are based on the International Standard Classification of Education (ISCED) and are classified as: *i)* low (ISCED 1–2, primary and lower secondary education), *ii)* middle (ISCED 3–4, upper secondary education) and *iii)* high (ISCED 5–6, tertiary education). Additionally, to estimate the weights of each education group, we obtained the population structure by sex, 5-year age groups and 5-year periods for each country from the Human Mortality Database (HMD) Human Mortality Database (2023).

We begin by discussing the case of Swedish females. Figure 3 shows the smoothed age-at-death distributions for Swedish females by educational level in two points in time, 1991-1995 and 2011-2015. The actual curves does not drop to zero on the right hand side, a pattern often encountered in low mortality countries with high life expectancy when data for the last age groups is aggregated at a not too old

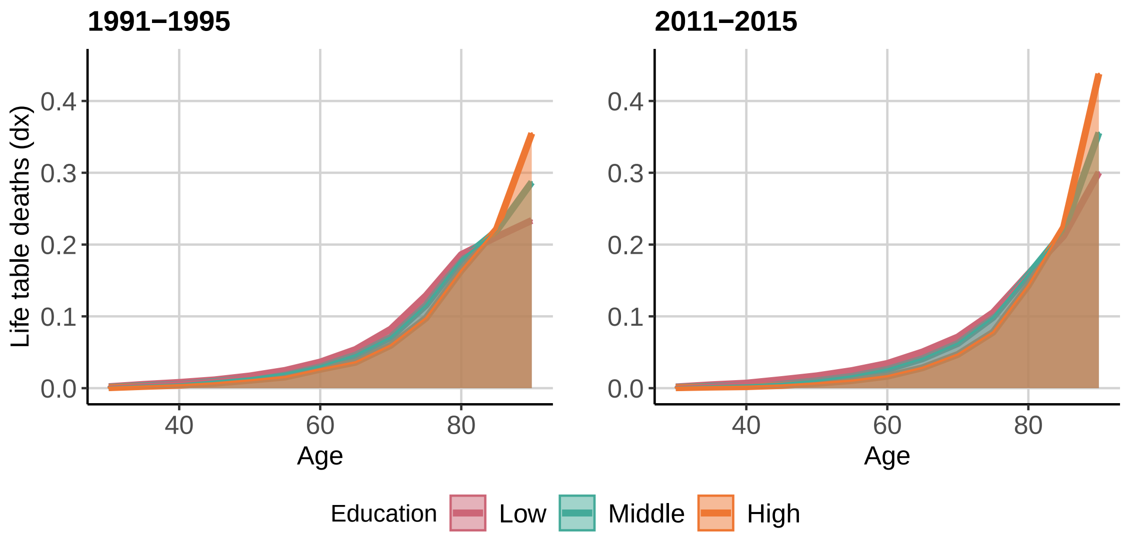

Figure 3: Age-at-death distributions for Swedish females by education level in 1991-1995 and 2011-2015, ages 30–90+.
Source: Authors' elaborations on data from Németh et al. (2021).

age, as in the present example. Regardless of this unusual shape in the age-at-death distribution, the PTV may still be estimated as the distributions still add to one.

Over time, age-at-death distributions for all education groups for Swedish females shifted to older ages. Between 1991-1995 and 2011-2015, life expectancy at age 30 increased from 51.2 to 52.3 for the low educated, and from 54.9 to 56.7 for the high educated. Lifespan variation, measured by the standard deviation of the ages-at-death, increased for the low educated (11.7 to 11.9), while it decreased for the middle and high educated, from 11.2 to 10.7 and from 10.9 to 9.7, respectively. Similar changes in mortality are observed for Swedish males and for Danish population during the study period, with life expectancy increases in all education groups and lifespan variation decreases only in the middle and the high educated groups.

Figure 4 shows the PTV at age 30 for Sweden and Denmark by sex alongside three commonly used measures of socioeconomic inequalities in mortality: the range of life expectancy at age 30, the range of lifespan variation at age 30 (measured with the standard deviation of the ages-at-death), and the slope index of inequality (SII) of the age-stadarised mortality rates (using the WHO World Standard population). Figure S2 in the Supplementary Materials shows the relative counterparts of these measures and the SII and RII for life expectancy and lifespan variation. We start by comparing the trends of these measures and then move on to analysing their levels.

For Swedish females, the range in life expectancy increased until 2006-2010, and then decreased afterwards. Conversely, the range in lifespan variation[2] increased throughout the study period, though it stagnated between 1996-2000 and 2006-2010. From 1991-1995 to 2006-2010 the range in life expectancy and the range

[2]Notice that lifespan variation is higher for the low than for the high educated, so to avoid negative values the range is inverted to the formula shown in Table 1. It is the difference in lifespan variation between the low and the high educated.

of lifespan variation had a relative changes of 25.7 percent and 91.4 percent, respectively. This difference further increased in the last period. The SII shows less pronounced changes, with a relative change of 7.8 from 1991-1995 to 2006-2010, and a negative relative change of -3.7 percent if we consider all the study period. Focusing on the last 5-year period, the inequality measures suggest different conclusions regarding socioeconomic inequalities in mortality: measures based on life expectancy indicate decreasing inequalities, while the range in lifespan variation indicates the opposite, making it difficult to determine whether inequalities increased or decreased. This is where the PTV can help to solve this dilemma, as it contains information from both positions and shapes of the distributions. The PTV started decreasing since 2001-2005, suggesting that equality increased (against evidence from the range in lifespan variability), and that the reduction of inequalities occurred before what the range in life expectancy and SII measures suggest. This example shows that patterns of convergence between groups may be hidden when looking only at summary measures of the age-at-death distribution such as the gap in life expectancy or lifespan variation.

The opposite situation may also be true. Patterns of divergence between distributions may be hidden by summary measures. This is the case for Danish men. The range in life expectancy and lifespan variation suggest that inequalities stagnated during the most recent period, relative changes of 0.9 and 1.5, respectively. The SII suggests that inequality reduced (relative change of -5.8). However, PTV shows a continuation of the divergence between groups (relative change of 4.6).

For Swedish males and Danish women, the PTV shows similar trends to the range in life expectancy and lifespan variation, with all measures reflecting increasing inequalities across groups. However the level of inequality of these two populations does change with the choice of measure. For measures based on life expectancy, the level of inequality for Danish females quickly surpasses that of the Swedish males, however in terms of the PTV, this takes longer and is only visible in the last period.

Focusing on the levels of inequality of the different measures, Danish males have the highest level of inequality throughout the study period, regardless of the measure. The lowest level of inequality is instead dependent on the measure used: according to the range and SII measures, Swedish females have been better positioned than their Danish counterparts throughout practically all studied periods; conversely, the PTV suggests that this occurred only in the most recent period. The most striking difference in the level of inequality between measures is indeed reflected in Swedish females. According to the range of lifespan variation, it has the lowest level of inequality through most of the periods. However, according to the PTV, it had the highest level of inequality amongst the studied populations in 2001-2005. This suggest that the PTV is incorporating information that is not reflected in the other inequality measures.

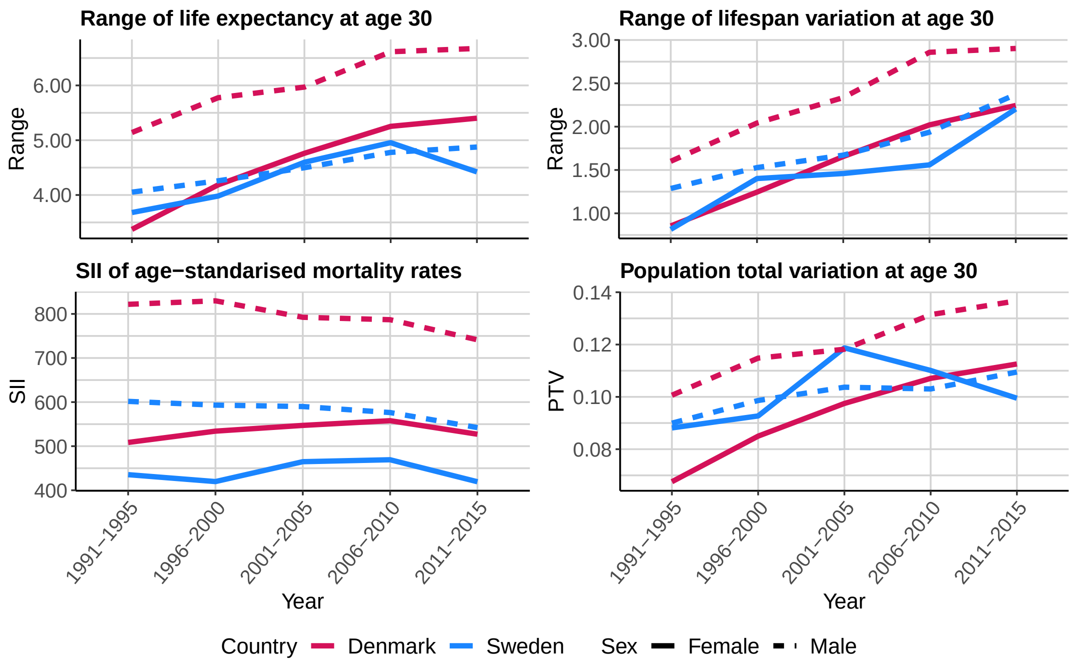

Figure 4: Trends in measures of inequality in mortality by sex for population groups defined by education level, Denmark and Sweden, 1991-1995 to 2011-2015. *Source:* Authors' elaborations on data from Németh et al. (2021) and Human Mortality Database (2023).

4.2 Mortality inequality by area-level deprivation index in England

Analysis of socioeconomic inequalities in mortality is often restricted to countries with high quality individual-level data, which allow to derive unbiased mortality estimates by population's subgroups (Kunst et al., 1998; Shkolnikov et al., 2007). Such data is available for a few countries and can suffer from issues such as increasing selection of some categories and changing composition (McCartney et al., 2017). In contexts where individual-level indicators of socioeconomic status are not available, it is possible to estimate area-based mortality indicators (Dukhovnov and Barbieri, 2021; Seaman et al., 2019b). These have the added advantage of providing estimates for the whole population, starting from age zero rather than from an older age. Given that area-based life tables can be derived, it is possible to estimate the PTV to measure inequality in mortality.

Here we present an example using the English Index of Multiple Deprivation (IMD) 2015 deciles (Smith et al., 2015). The IMD is the official measure of relative deprivation for small areas in England. It combines information from seven domains: income, employment, education, health, barriers to housing and services, crime and living environment. It ranks 32,844 small areas, with roughly the same population, by level of deprivation. The areas are then grouped into deciles, with each decile containing 10 percent of the small areas. We use the life tables by IMD decile estimated for England by UK's Office for National Statistics (Office for National

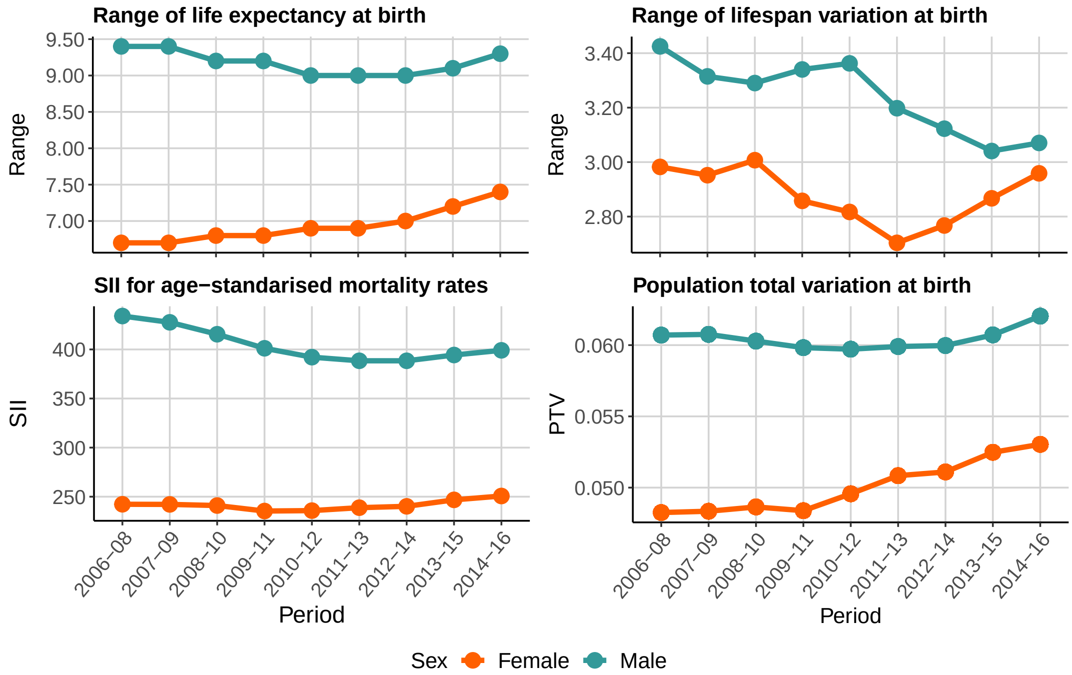

Figure 5: Trends in measures of inequality in mortality by sex for population groups defined by area-level deprivation deciles, England, 2006/2008-2014/2016.
Note: The range of the lifespan variation is inverted, it is the difference in lifespan variation between the least and the most advantaged deciles.
Source: Authors' elaborations on data from Office for National Statistics (2018).

Statistics, 2018). These are single-age life tables by sex starting at age zero. The estimates are based on mortality rates calculated for a three year periods from 2006-2008 until 2014-2016.

Figure 5 presents the PTV for groups defined by deprivation deciles in England for the period 2006-2008 to 2014-2016 by sex. Alongside we include the same three measures of socioeconomic inequalities in mortality used in the previous example. In this case the life expectancy and the lifespan variation are calculated at birth. Figure S3 in the Supplementary Materials shows the relative counterparts of these measures. Again we start by discussing trends and then levels of inequality.

For females, the PTV remained stagnant from 2006-2008 until 2009-2011, reflecting that mortality inequality between deciles did not change. After this period, a steady increase started and continued until 2014-2016. During the complete period, the PTV had relative increase of 9.7 percent. This trend differs to the ones portrayed by both the range of life expectancy and of lifespan disparity[3], which show either a continuous increase for all the study period or rather an erratic decrease followed by an increase in more recent years, respectively. Contrastingly, the SII shows and almost constant trend, detecting no change in inequalities between deciles, with a relative change of 3.7 percent during the studied period. Again, this suggests that the PTV is exposing distributional similarities hidden by summary measures.

[3]As for Figure 4, we invert the formula shown in Table 1 to avoid negative values of the range.

In the case of males, both trends, range of life expectancy and PTV, show similar patterns, with a slow decrease in the first years followed by an increase in the later years. However, the direction of the relative change from 2006-2008 to 2014-2016, differs by measure. According to the range in life expectancy, there was a relative decrease of 1.1 percent in inequality, while the PTV points to a relative increase of 2.1 percent. Similarly, the SII decreases at the beginning, however it shows no sign of worsening inequalities at the end of the period. Its relative change was of -10.3 percent for the whole period. The range of lifespan variation decreases erratically over time.

All four measures of inequality show similar results when comparing the levels of inequality by sex. Males have higher inequality in mortality than females, with the gap reducing by the end of the study period. The most notable reduction in the difference in inequality levels by sex is observed in the range of lifespan variation in 2014-2016, with a relative change in the gap between sexes of -74.8 percent. Compared to the SII, the PTV shows a greater reduction in the gap in the level of inequality between both sexes, with a relative change of -27.3 percent compared to -22.9 percent for the gap in SII.

5 Discussion

The aim of this paper was to evaluate whether measuring distributional differences between age-at-death distributions provides new and additional insights on socioeconomic inequalities in mortality than conventional summary-based measures. For this purpose, we presented a measure, which we call Population Total Variation (PTV), that captures distributional differences between two or more population's subgroups, thereby allowing us to refine the measurement of socioeconomic inequalities in mortality.

Our findings show that the PTV can uncover inequality trends that may differ from those obtainable with standard methods based on summary measures of mortality. This is in line with previous research measuring distributional similarity in age-at-death distributions (Shi et al., 2023). For example, in the case of females grouped by area-level deprivation in England, the PTV reveals a sharper increase in mortality inequalities between deciles than what is reflected by other measures. One possible explanation of the different trends is that the PTV reflects both shifts and compressions in the age-at-death distributions. In the study period, the age-at-death distributions of all deciles are shifting to older ages and are compressing at older ages, however the lower deciles are doing so in a lower speed than the higher deciles, which results in increasing inequalities over time.

Furthermore, it is not uncommon that inequality measures based either on life expectancy or lifespan variation may point in opposite directions, making it to difficult to draw a conclusion, as in the case of Swedish females between 2006-2010 and 2011-2015. In such instances, the PTV can solve this dilemma, since it incorporates both measures along with all moments of the group-specific distributions. During this period, the underlying distribution of high-educated Swedish females had an increase in mortality between ages 80 and 89, which caused a decrease in life expectancy, a sharp decrease in the lifespan variation, and slight increase in its age-standardized mortality rate, reflected in the trends shown in 4. The age-at-death distribution of the high educated moved closer to those of the other groups, which was captured by the decrease in the PTV.

In other cases, for example that of Swedish males and Danish females, the trends in the PTV may not differ from that of other measures of inequality. However, as seen in that example, the level of the inequalities reflected by each measure may still be different. Regardless of the case, we believe that it still useful to employ the PTV, because it (i) captures additional information on the changes of the age-at-death distributions, and (ii) summarizes the underlying changes in a single measure, thereby allowing users to directly evaluate changing inequalities.

For the two examples shown in this paper, changes in the PTV are mainly driven by mortality changes in the lower and upper groups (high educated and richest decile), and minimally affected by mortality changes in the intermediate groups. This occurs because the distribution of the intermediate groups is contained by those of the extreme groups, therefore it is inside the overlapping area. This is however a specific feature of these applications, and is not a general rule (for example, in the case of non-ordered categories, such as race; see also Figure 1).

The PTV has several advantages compared to other measures of inequality in mortality. As previously mentioned, recent propositions in demography using age-at-death distributions to study mortality inequality have limited the analysis to pairwise comparisons (Sasson, 2016; Shi et al., 2023; Vaupel et al., 2021). Our approach extends this line of research by proposing a measure that does not rely on performing all pairwise comparisons, which may problematic when a large number of groups is studied. For example, for the case of England, measuring inequality with pairwise comparison would require computing the same metric 45 (i.e., $n(n-1)/2$) times, and then performing some averaging. For settings with larger number of groups, the comparisons would quickly become onerous. Conversely, our measure provides a more elegant and straightforward approach to consider various groups.

Compared to other measures, the PTV is simple to compute and to interpret. The data requirements of the PTV are the same as the ones needed to estimate life expectancy and lifespan variation by socioeconomic groups, as it is estimated from the life table death distribution. Unfortunately it cannot be estimated in contexts where life tables by socioeconomic groups are unavailable. To go around this limitation, we presented an application on how the PTV may be used for area-level socioeconomic measures, which may available in contexts where individual-level data on socioeconomic position is not available. We hope that this measure encourages more statistical offices to produce life tables by different socioeconomic characteristics.

Recent developments have focused on decomposing the total lifespan inequality of a population into between- and within-group contributions (Permanyer et al., 2023). The PTV goes beyond this line of research as it is not a measure of overall lifespan inequality, but rather of the overall similarity across multiple distributions. It could be pointed out that, although our main goal is not to measure total lifespan inequality, the PTV implicitly accounts for this by comparing the full age-at-death distributions; as such, it is an indicator of inter-group inequalities, while taking into account inter-individual inequalities. Additionaly, it should be explicitly mentioned that this distributional approach supports the argument that in health inequalities, what is morally significant is the systematic association between health and socioeconomic status (Asada, 2013).

The PTV has some potential limitations. Firstly, it does not indicate the direction of the distributional convergence. For example, if the PTV decreases, it is not possible to distinguish if it is because mortality decreased for the worst-off group or because it increased for the better-off group. A possible solution is to extend the measure to include some population-level central tendency measure that would help contextualize the PTV according to the total population's mortality levels. Following Wagstaff (2002)'s overall measure of health *achievement* and the UN's Inequality-adjusted Human Development Index (IHDI) (United Nations Development Programme, 2022), we suggest using an Inequality-discounted Mortality Index (IDMI) at age x: $\text{IDMI}_x = e_x * (1 - \text{PTV}_x)$, where e_x is the remaining life expectancy at age x for the total population and PTV_x is the population total variation at age x. The IDMI is a mortality index that combines information on the population-level life expectancy and inequality across socioeconomic groups. Specifically, the population-level life expectancy – or any other central tendency measure of interest – is reduced according to the level of distributional differences between subgroups measured by the PTV. The measure is similar in spirit to the IHDI, which discounts life expectancy by a measure of lifespan variation. If all subgroups have the same age-at-death distribution, then the PTV is zero and the IDMI is the life expectancy. On the contrary, if the subgroups distributions never overlap, then inequality is at its maximum and the value of the index will be zero. For simplicity we don't include it here, but one may extend the IDMI by including a weighting parameter that allows users to decide the relative importance between reducing socioeconomic inequality and overall mortality improvements, accommodating their subjective perspectives on inequality.

Additionally, the PTV does not incorporate information on the share the population in each group. This has been pointed out as a desirable property in the context where strategies to decrease health inequalities sometimes focus on the social determinants, for example promoting education (Renard et al., 2019). Users of the PTV should be aware that this omission implies that what is important is the group, regardless of its size (Keppel et al., 2005). Future studies should look at extending the PTV to incorporate population weights. Notice that on the example by area-level deprivation index the choice of weighting does not affect the results as all groups are roughly the same size.

Furthermore, policy interventions are typically based on real populations rather than on synthetic cohorts of the life tables (Dudel and van Raalte, 2023). As the PTV is estimated from life table age-at-death distributions, it might not directly highlight where policy efforts should be primarily concentrated. Nonetheless, we believe that the PTV can still inform policies by providing additional information than commonly used measures of inequalities, and by indicating the relevant age-groups of the group-specific life table populations where mortality differences occur. Future work will be devoted to derive structure-adjusted PTV measures, as recently proposed for other mortality measures (Pifarré i Arolas et al., 2023).

6 Conclusions

In this paper we presented the population total variation (PTV), a measure of overall similarity across distributions. The PTV can be considered as an alternative measure of inequality in mortality that goes beyond the first two summary measures of the distribution. Our results highlight that the PTV overcomes potentially conflicting results that may arise from looking at the two summary measures separately.

We believe that the measurement of socioeconomic inequalities in mortality may be refined by using the whole information of the group-specific age-at death distribution, and hope that this article contributes to the case of using the age-at-death distributions when comparing population subgroups. This is particularly valuable in a context where mortality improvements are becoming less homogeneous between populations and changes may not be reflected by summary measures.

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