Variable-r Projection

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Abstract

Population projections remain among the core demographic undertakings. The Cohort Component Method (CCM) is the established procedure that combines basic demographic components (survival, fertility, and migration) information to determine the future population. However, the three basic demographic components need first to be forecasted by age and sex to then be included in the CCM. Here we propose using an alternative measure as the primary forecasting component, namely the age-specific growth rates. For each cohort the trend over time and age of those age-specific growth rates (or *variable-r*) remain unchanged from birth to older ages (around age 70, varying from country to country). Thus, the future age-specific growth rates for those who are below age 70 and present at the baseline moment of projection, are already determined by their current age-specific growth rates. For the rest of the age-groups the complementary fertility and mortality forecasts can be included to obtain forecasts of the population at those ages.

Introduction

UN World Population Prospects estimates that the world's population will continue growing from 7.7 billion today to a peak at nearly 11 billion around 2100 (UN, 2022). UN and many other international as well as national institutions rely on the Cohort Component Method (CCM) to obtain future counts of population by age and sex (Preston et al. 2001). The CCM includes as input future trends of population components (fertility, mortality and migration). Thus, the first step in applying the CCM is to obtain reliable future estimates of those population components. In this study, we explore a fourth population component that retains a lot of its cohort information from birth, namely the population age-specific growth rates.

Age-specific growth rates are determined by the contribution of historical changes in fertility, mortality and migration. The *variable-r* model separates the past contribution of each of these demographic components from the age-specific growth rates (Arthur and Vaupel 1984; Lee and Zhou 2017; Murphy 2017; Preston and Coale 1982; Preston and Stokes 2012; Preston and Vierboom 2021). Traditional *variable-r* methods require extremely long demographic series (birth, mortality and migration) to explore growth at all ages, with particularly high data demands at oldest-old ages. However, strong consistency is found between age-specific growth rate data up to adult ages (varying around age 70 from country to country), and their corresponding cohort growth rate at birth (Canudas-Romo, Shen and Payne 2022). This finding has allowed to suggest for shorter series of data to estimate the components of the age-specific growth rates (Canudas-Romo, Shen and Payne 2021). These strong cohort relations are used here to project population counts based on their age-specific growth rates and its components.

Data, Methods & Preliminary Results

Our analysis uses age- and sex-specific population counts and death rates for national populations from the Human Mortality Database (HMD).

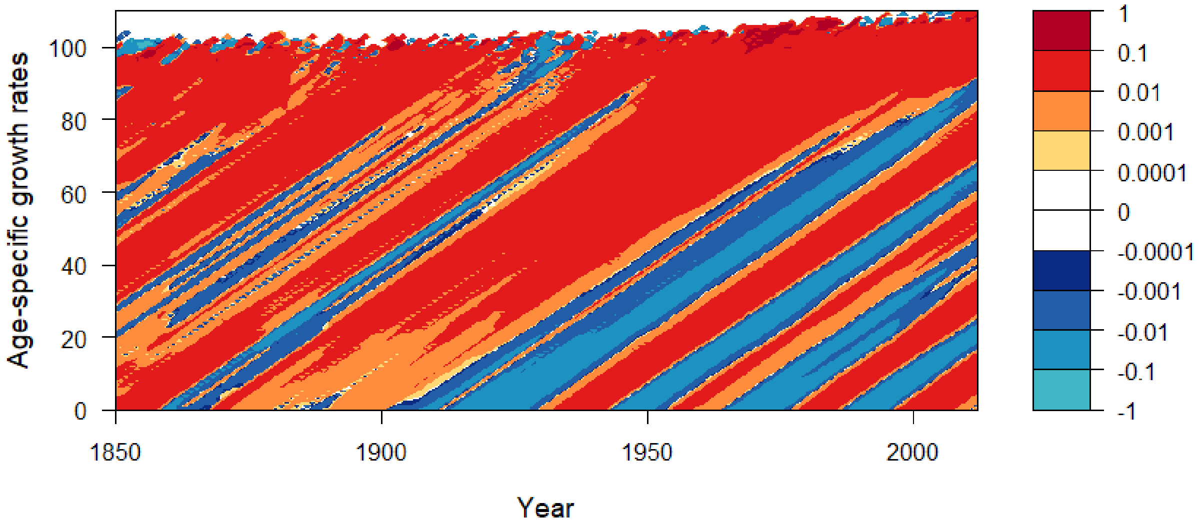
Let a dot on top of a variable correspond to derivatives of demographic variables with respect to time (Vaupel and Canudas-Romo 2003). The age-specific growth rates at age x and time t, denoted $r(x,t)$, are defined as the relative change in population counts, $P(x,t)$, as

$$r(x,t) = \frac{\dot{P}(x,t)}{P(x,t)} \,. \tag{1}$$

Details of the estimation of derivatives are found in the appendix.

Figure 1 presents the age-specific growth rates for Swedish females from 1850 to 2012. The remarkable cohort trends in this Figure are evident for both, big birth-cohorts (e.g. those with growth rates above 1%, i.e. increasing were born in 1850-57, 1869-71, 1932-42, 1955-58, 1979-85, and 1997-2007) and small cohorts (e.g. those with growth rates below -1%, i.e. decreasing: 1859-61, 1907-10, 1913-27, 1944-51, 1965-69, 1971-75, and 1988-95). This strong cohort pattern is remarkable because growth rates are calculated between two years (period) and fixing ages, meaning not from a cohort perspective. If we can triple the use of the word remarkable, it is also interesting that this growth level remains at its growth rate at birth level until advanced ages, around age 70.

Figure 1. Lexis surface of female age-specific population growth rates for Sweden.



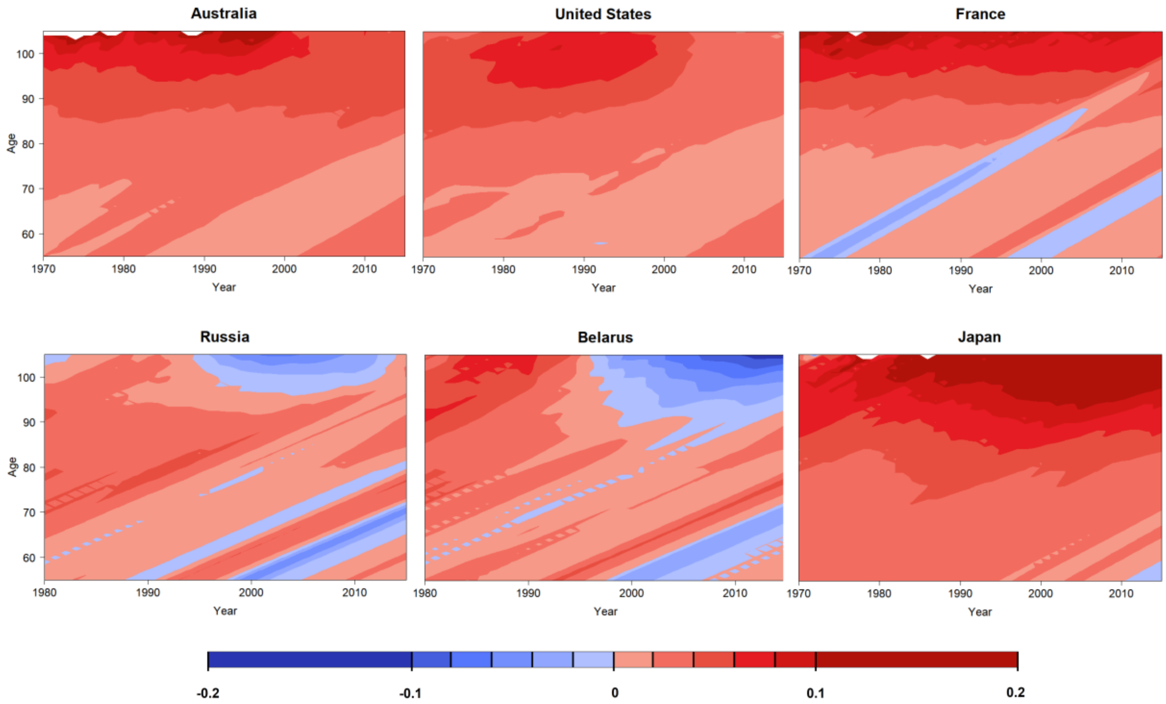
Source: author's calculations based on the HMD (2023).

Although, the Scandinavian nation is known for its reliable and long historical demographic trends, the strong cohort similarity between age-specific growth rates and their past cohort rates is observed across other low-mortality nations with shorter times series. Figure 2 includes a Lexis surface of the age-specific growth rates for Australia, Belarus, France, Japan, Russia, and the United States at ages 55 and over. The cohort trends observed for Sweden, are repeated in all these countries even to advanced ages. Although this is stronger in France, Russia and Belarus, than in Australia, Japan and the USA, for all countries cohort trends in their age-specific growth rates are observed.

This strong cohort pattern can be explained by looking at the *variable-r* components. The age-specific growth rates in Eq. (1) can be decomposed to include information on growth rate at birth, and the change over time in the probability of survival and migration between birth and age *x* (Horiuchi and Preston 1988, see Figure A1) as

$$r_x(t) = r_B(t - x) + \Delta S^{t-x} + \Delta M^{t-x}, \qquad (2)$$

Figure 2. Lexis surface of female age-specific population growth rates from ages 55 to 105 for Australia, Belarus, France, Japan, Russia, and the United States.



Source: author's calculations based on the HMD (2023).

where $r_B(t-x)$ denotes the growth rate at birth and $\Delta S^{t-x} = \frac{\dot{S}^{t-x}(x)}{S^{t-x}(x)}$ and $\Delta M^{t-x} = \frac{\dot{M}^{t-x}(x)}{M^{t-x}(x)}$ are the terms that compare the change in probabilities of surviving and net migration for the birth cohort *t-x* (see Figure A1 for a schematic representation for the *variable-r* components).

Figure 3 shows the age-specific growth rates and its components as defined in Eq.(2) for Swedish females from 2008 to 2018. The age pattern of the growth rates $r_x(t)$ is defined by booms (cohorts reaching ages 65-75, 20-30, 0-10, and to a lesser extent 45-52, in 2008) separated by busts (cohorts reaching ages 80-90, 55-65, 32-42, and 12-20, in 2008). Another important characteristic of the trend is that age-specific growth rates mimic the growth rates at birth of their corresponding cohorts, or $r_B(t-x)$, except at older ages where improvements in mortality take over as the explanatory factor of the growth rates.

This strong relationship between age-specific growth rates and their cohort growth can be used to estimate population counts in the future. Preston and Vierboom (2021) have applied such rates to the US population and obtained future population trends. Figure 4 presents the same concept applied to population counts of Swedish females. In this Figure it can be seen how the population counts progress over age, particularly observable for those big and small cohorts.

Figure 3. Female age-specific growth rates and its components: mortality (dS), net-migration (m_i) and growth rate at birth (r_B) as in Eq. (2), for Sweden from 2008 to 2018.

Source: author's calculations based on the HMD (2023).

Discussion and Future steps

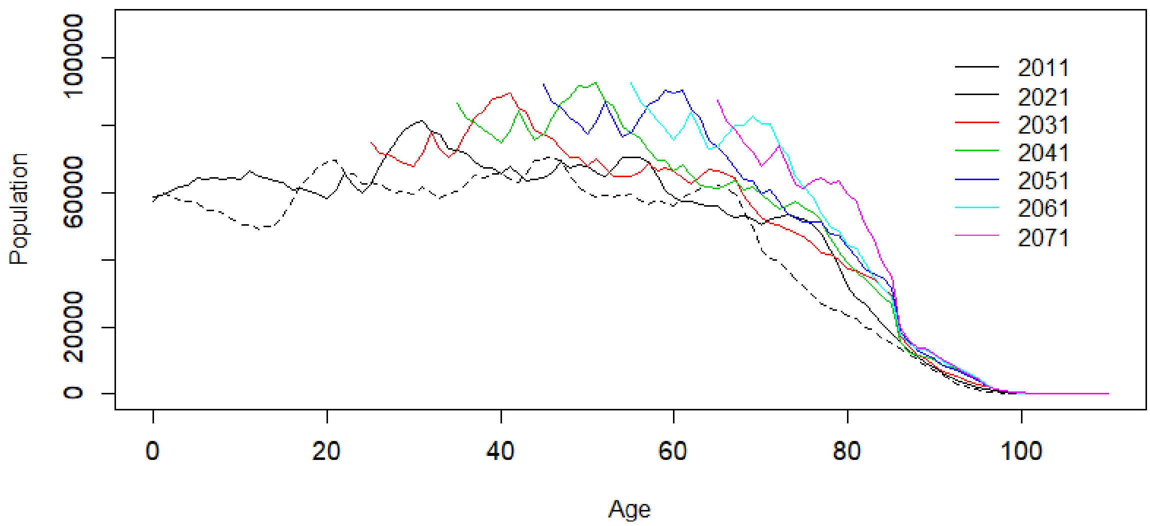
This simple procedure of projecting population counts only requires age-specific growth rates with input from two years of information, for example based on two censuses. For countries that lack historical data or with incomplete vital-statistics systems to create trends of mortality, fertility, and migration into the future, age-specific growth rates from censuses can be used for population forecasting. Similarly, subnational populations with shorter time trends of information could also benefit of such method. However, the assumption of similarity in age-specific growth rates over age will need to hold.

However, the relations highlighted in the *variable-r decomposition* in Eq(2) have not been used. Thus, several further improvements to the *variable-r forecasting* will be included for the European Population Conference 2024:

1) For every new year of forecasted data, forecasted age-specific fertility rates will be applied to the population counts to obtain the new baby counts and thus younger cohorts.
2) Mortality which is a dominant component in old age *variable-r decomposition* can be further incorporated in the current variable-r projections to obtain a better estimation for population counts than the current straight age-specific growth rates application. This is the case, since growth rate of younger ages and improvements in mortality compose the old age growth rates (Canudas, Shen and Payne 2021).

3) Our results will be compared with those from the UN projections, and historical data will be used to compare the accuracy of the *variable-r* projection vs the CCM.

Figure 4. Projection of the female Swedish population in 2021 (and historical 2011) to 2071 based on the population present in 2021 and the age-specific growth rates from 2011 to 2021.



Source: author's calculations based on the HMD (2023).

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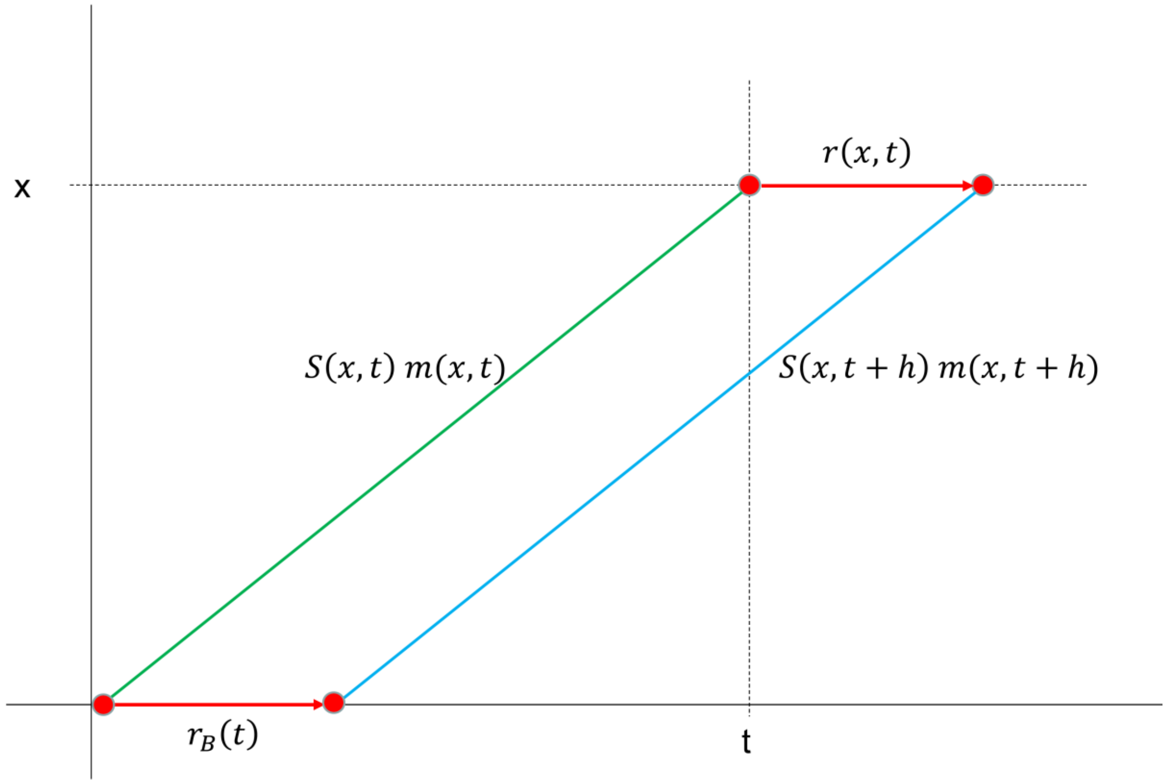
Appendix

Figure A1. Lexis diagram representation of the elements of the *variable-r* decomposition



Approximating Continuous Change

Our equations in the main text, are in continuous, however data is found only annually and approximations are needed for: the relative derivative, midpoint calculations, and derivatives. We implemented standard approximations for those terms (Vaupel and Canudas-Romo 2003; Preston et al. 2001).

Given a demographic function *v(x,t)* measured at two time points *t* and *t+h*, we approximated the relative derivative assuming a constant rate of change over time as

$$\frac{\dot{v}(x,t+h/2)}{v(x,t+h/2)} \approx \frac{\ln\left[\frac{v(x,t+h)}{v(x,t)}\right]}{h}. \tag{A1}$$

We approximate the midpoint for a function using

$$v(x,t+h/2) \approx [v(x,t)v(x,t+h)]^{\frac{1}{2}}, \tag{A2}$$

and the derivative with respect to time by

$$\dot{v}(x,t+h/2) = \left[\frac{\dot{v}(x,t+h/2)}{v(x,t+h/2)}\right] v(x,t+h/2). \tag{A3}$$