Developing male fertility forecasts to inform kinship forecasts

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1 Topic and theoretical focus

The ability to forecast the ways in which kinship networks are likely to evolve in the future provides a rich framework that can be used to answer many policy-relevant questions. Examples include the availability of informal care through the kin network, and the impact of different aspects of demographic change on social security.

The matrix formulation approach to modelling kinship first proposed in Caswell (2019) and extended in subsequent papers, provides an elegant and computationally efficient way to quantify many aspects of the kinship network, not least age structures and expected numbers of kin. While only female kin through female lines of descent were considered in previous kinship models, the matrix model of Caswell easily extends to accommodate two sexes (Caswell, 2022). Male fertility tends to be less well documented, with data that *is* collected generally having lower quality due to a comparatively high proportion of missing paternal ages (Dudel and Klüsener, 2021; Schoumaker, 2019). For this reason, where male fertility rates are not available, Caswell (2022) investigates an "androgynous approximation", i.e. the use of female rates for both sexes. It performs reasonably well when compared with the two-sex model, but it is clearly preferable to use the true male fertility rates where possible. Key differences between male and female fertility rates include the later transition to parenthood for males and their ability to have children at much later ages compared to females (Schoumaker, 2019).

The framework of Caswell also extends to include time-varying rates (Caswell and Song, 2021), enabling projections of kinship structures to be produced. These kinship forecasts require not only estimates of observed demographic rates for both sexes, but also projections of these rates. While existing mortality forecasting methods tend to consider males and females in tandem, for example by applying the proposed model to combined rates (e.g., see Lee and Carter (1992)) or by developing a joint model (e.g., see Wiśniowski et al. (2015)), fertility forecasting has thus far focused almost exclusively on females (e.g., see Ellison et al. (2020)). This is not surprising given that population projections, which are one of the primary outputs informed by fertility projections, only require forecasts of female fertility to give the projected number of births in each future year. In line with the androgynous approximation above, these projections could simply be used for males (see Alburez-Gutierrez et al. (2023) for an example of this), but more accurate inferences can be obtained for future populations if, where possible, we include male fertility rates and forecast them.

This brings us to our aim, which is to develop a model to forecast male fertility rates. Just as with forecasts of female fertility rates, this is of substantive interest in and of itself. However, a key secondary use of the forecasts is to inform kinship projection models. We work within a Bayesian framework to appropriately quantify uncertainty, and demonstrate our approach for two countries with good quality male fertility data, namely England & Wales and the USA.

2 Data and methods

A set of estimates of male age-specific fertility rates (ASFRs) across 17 high-income countries is available from the Human Fertility Collection (Human Fertility Collection, 2023). Dudel and Klüsener (2019, 2021) give a detailed description of the underlying data and methodology and provide a comprehensive analysis of this rich data source. We adapt the accompanying code (Dudel, 2020) for England & Wales and the USA to obtain annual estimates of the number of births by single year of age of father, which takes the values $15, 16, \ldots, 54, 55+$ for England & Wales and $15, 16, \ldots, 58, 59+$

for the USA[1]. We choose these countries due to the reasonably long time series of births that are available (1982-2016 for England & Wales, and 1969-2021 for the USA), and the ease of accessibility of the raw birth counts (from ONS (2018a,b) and NBER (2023)). We source population denominators from the Human Mortality Database (Human Mortality Database, 2023).

For each country, initial investigations have focused on modelling the true male age-period fertility rates as a smooth, non-parametric two-dimensional (2D) age-period surface, which we can then extrapolate to future years. We estimate this surface using a P-spline approach (Eilers and Marx, 1996), which has also been applied in the context of mortality forecasting (Currie et al., 2004; Camarda, 2019). Under this approach, the smooth surface is expressed as a linear combination of basis functions, which are themselves smooth 2D functions of age and period. These basis functions are constructed as the product of B-splines, which are smooth curves that are non-zero for only a small part of the covariate range. Smoothness is achieved by penalising the first-order differences between the coefficients of consecutive basis functions in both dimensions, i.e. across age and period, simultaneously. We assume that the observed birth counts follow a negative binomial distribution in order to account for overdispersion relative to the Poisson distribution.

More precisely, for age a and year y, we let b_{ay} be the observed birth count, N_{ay} be the population exposure to risk, and θ_{ay} be the true fertility rate. We assume that $b_{ay} \sim \text{NegBin}(N_{ay}\theta_{ay}, \phi)$, where $N_{ay}\theta_{ay}$ is the expected number of births and $\phi > 0$ is the dispersion parameter. We then express the logarithm of the true rate as the product of the row vector $\mathbf{X}_{ay}$, which contains the value of each basis function for this age and year, and the vector $\boldsymbol{\beta}$ of coefficients, i.e. $\log(\theta_{ay}) = \mathbf{X}_{ay}\boldsymbol{\beta}$. We take the male reproductive age range to be 15-69, and so for the open-ended age interval starting at age $a_{\max}$, we assume that $b_{(a_{\max}+)y} \sim \text{NegBin}(\sum_{\alpha=a_{\max}}^{69} N_{\alpha y}\theta_{\alpha y}, \phi)$. We fit the model using the `rstan` software package (Stan Development Team, 2023), which enables efficient sampling from Bayesian models.

To improve forecast plausibility and to reflect our prior knowledge that aggregate measures such as the total fertility rate (TFR) change reasonably slowly over time, we investigate the application of a constraint on the TFR. To assess the predictive accuracy of our proposed model, we compare our forecasts with some of the current best-performing models in the female fertility forecasting literature (see Bohk-Ewald et al. (2018) for a recent review). We also investigate the joint forecasting of male and female fertility, borrowing strength across both sexes (see Section 1 in relation to mortality).

3 Preliminary results

We present preliminary findings for England & Wales and the USA, fitting our proposed model to data from 1982-2011 and forecasting to 2020. Figure 1 displays the observed ASFRs (panels 1 and 3) and the posterior median ASFRs from the smooth age-period surfaces (panels 2 and 4). We note that the observed rates are available up to 2016 (England & Wales) and 2020 (USA). The proposed model provides a good approximation to the training data, with the smoothness assumption having the strongest impact at ages 25-35. The upticks in the observed rates for the open intervals at age 55 (England & Wales) and 59 (USA) are plausibly spread across the age range as we see a smooth decline to zero. We forecast continued declines roughly up to age 35, with rates above age 35 predicted to remain more stable. For the USA, these declines look more extreme than what was actually observed.

Figure 2 allows us to assess the forecasts in more detail, plotting the corresponding 95% prediction intervals for some selected years. This confirms the close fit up to 2011, with forecast performance strong for England & Wales but poorer for the USA, particularly in 2016 where the observed rates are close to the upper bound of the interval. By 2020 there is only a slight underprediction by the model.

[1] We note that Dudel and Klüsener (2019) take age 59 to be the upper bound for their rate estimates. Hence, for England & Wales, a smoothing method is used to distribute the births in the 55+ category across ages 55-59, which also slightly adjusts the counts at the preceding ages. We choose not to do this and instead model the 'raw' counts, i.e. the counts that have only been adjusted by imputing missing paternal ages conditional on the maternal age, directly.

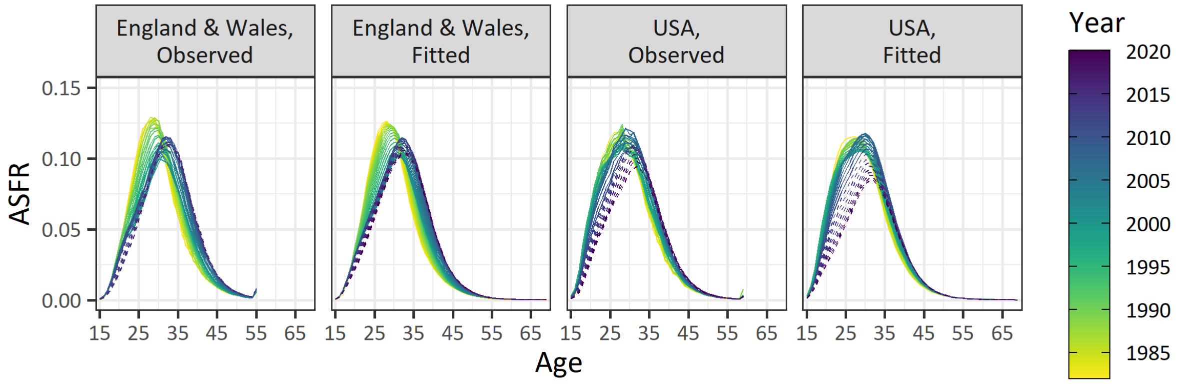

Figure 1: Observed age-specific fertility rates (ASFRs) from England & Wales and USA data (panels 1 and 3); posterior median ASFRs from the proposed model fitted to the corresponding rates (panels 2 and 4). Dashed lines indicate age-period combinations after 2011.

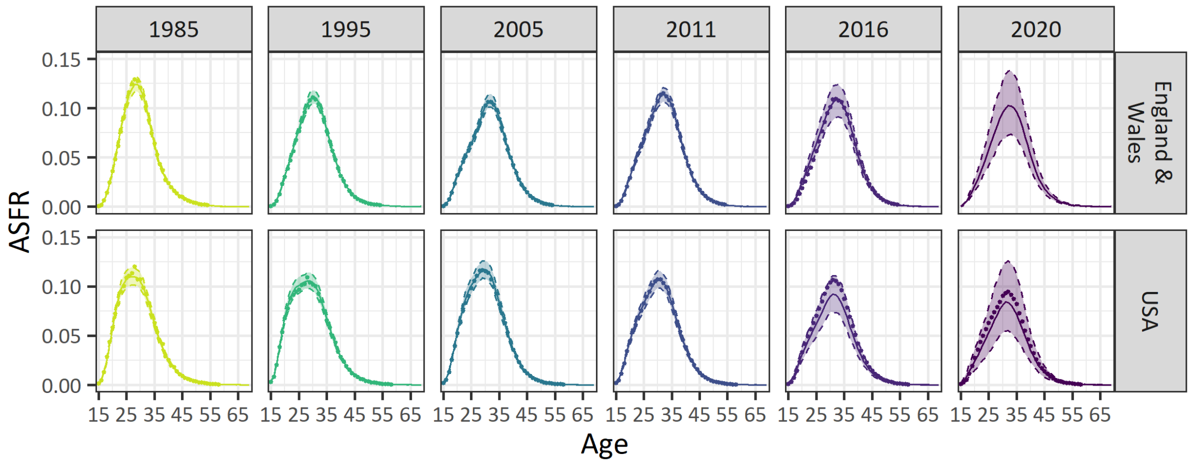

Figure 2: Age-specific fertility rate (ASFR) posterior medians and 95% prediction intervals for selected years, from the proposed model fitted to England & Wales (top row) and USA (bottom row) data. Corresponding observed values are overlaid as points. The years after 2011 are forecasts.

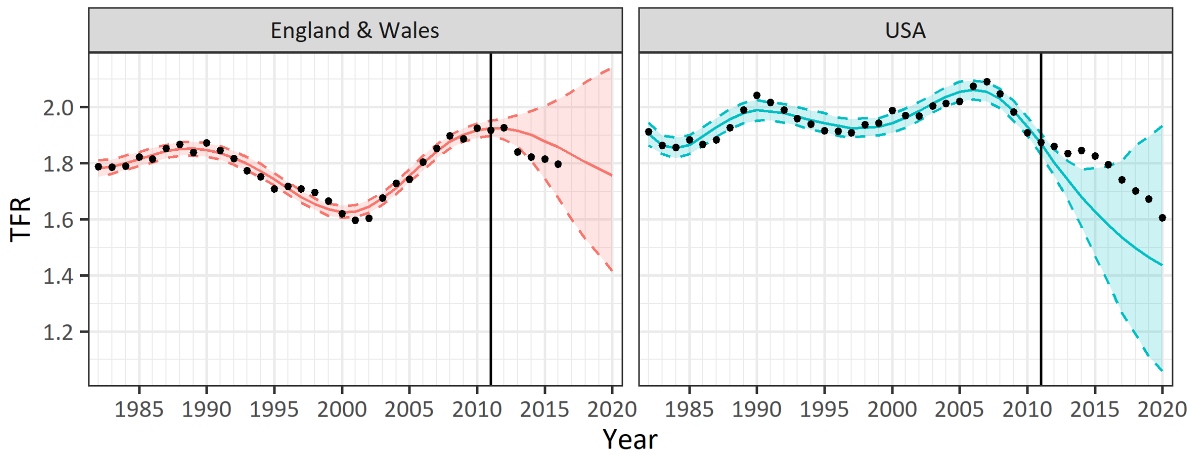

Figure 3: Total fertility rate (TFR) posterior median and 95% prediction interval from the proposed model fitted to England & Wales (left) and USA (right) data. Black points are observed TFR values. Vertical lines indicate the last year of data included in model fitting (2011).

Lastly, in Figure 3 we plot the posterior distributions of the TFR, i.e. the average number of children for a man experiencing the rates of a given year. This makes it easy to see that while we underestimate the speed of the future fertility decline for England & Wales, we overestimate it for the USA. These contrasting findings are likely caused in part by the differing trends evident as we approach the last year of the training data (2011). However, we also observe the closeness of the England & Wales posterior median to the future observed values, as well as the decreasing forecast errors over time.

4 Expected findings

Our proposal to develop a Bayesian model to forecast male fertility advances the current fertility forecasting literature which focuses on female fertility. The use of the resulting forecasts as an input to kinship projection models also extends the existing literature on this topic, by enabling more tailored sex-specific forecasts of kinship networks to be produced. Our findings have the potential to lead to improvements in the accuracy of kinship forecasts, therefore increasing the reliability of the resulting evidence for care policy that such projections can provide.

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