Incorporating Duration Dependency in Healthy Life Expectancy: How serious is the bias?

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Abstract

Demographic studies on healthy life expectancy mostly rely on the Markov assumption and suffer from the limitation that the duration of exposure to risk is not considered. There are models designed to account for duration dependency, such as the semi-Markov model and the multistate life table with duration dependency (DDMSLT). However, these models cannot be directly used on left-censored survey data as they require knowledge of time spent in the initial state, which is rarely known due to the survey design. This study proposes a flexible approach to utilize this type of survey data in a DDMSLT framework to estimate the multistate life expectancy. The approach involves dropping some left-censored observations but keeping as many as possible by the truncation of a duration length after which duration dependency is negligible. We apply this approach to older adults in the US based on the Health and Retirement Study to compute healthy life expectancy and examine the duration dependency compared to the typical multistate life table with Markov assumption. Our findings suggest that duration dependency is present in transition probabilities. However, the effect on healthy life expectancy is averaged out between the short-term states and the long-term states. As a result, the bias is minimal in the context of this study, and for the simplicity of the model, the Markov assumption is justified when calculating healthy life expectancy.

Keywords: Markov assumption, duration dependency, multistate model, longitudinal survey data, healthy life expectancy

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Introduction

The discrete-time multistate life table (MSLT) is widely used to calculate healthy life expectancy (HLE) in demographic and population health research. MSLT assumes that health transitions follow a first-order Markov Chain, where transition rates are assumed to depend only on the starting state. However, in reality, a person who has been disabled for five years would be unlikely to have the same probability of recovery as the person disabled for just one year. This what is known as duration dependence – the notion that the length of time a person has spent in any given state (e.g., in a state of disability) is an important factor or proxy factor influencing the chances of shifting out of the state. Various approaches have been developed over the years to relax the Markov assumption and incorporate duration dependence (e.g., Steele et al. 2004; Wolf 1988) and several studies across social, demographic, health and economic contexts have indeed pointed to the importance of duration times (Belanger 1989; van den Berg & van Ours 1996; Cai et al. 2008; Crowther & Lambert 2014; Maddox et al. 1994; O'Donnell 2021). In studies of health and disability, Maddox et al. (1994) suggested that the risk of impairment was time-dependent after controlling for demographic and socioeconomic status. Cai et al. (2008) also found that the probability of recovering from a disability decreased with the duration of the disability, while the probability of developing a disability decreases with a longer duration in active health. Incorporating duration dependence into demographic life table-based models though can be data intensive and computationally difficult, meaning that studies that incorporate duration dependence are very rarely able to produce estimates of multistate life expectancy. Using data on disability transitions in the United States, this study develops and tests options to address this gap.

Most studies exploring duration dependency only present transition probabilities without examining the impact on the HLE. There are several potential explanations for this gap, though a leading candidate is the methodological complexity in calculating multistate life expectancies with duration-dependent models. Specifically, the very large expansion in the state space of many duration-dependent models can make it infeasible if not near impossible to calculate life expectancies with a standard Markov life table. As a result, papers calculating HLE typically rely on the standard Markov MSLT and only mention the restrictiveness of the Markov assumption as a limitation. However, demographers have developed approaches for introducing duration dependence to multistate analyses, with particular applications to marital transitions (Belanger 1989; Schoen 2021; Wolf 1988). The MSLT with duration dependence (abbreviated to DDMSLT) has been discussed since the 1980s after the development of a discrete-time semi-Markov approach (e.g., Littman & Mode (1977); Hennessey (1980)). Wolf (1988) proposed a generalized multistate life table depending upon the duration of risk exposure and showed that DDMSLT, though inherently a non-Markovian process, embeds the Markovian component. Thus, DDMSLT shares most properties with MSLT, except that the state space is substantially larger because the states are duration-category-specific.

One of the key inputs required for designing multistate models that incorporate duration dependence is information on how long people have been living in their present state. When the start time of an event and state transition is known, like the timing of a job loss and entry into unemployment, semi-Markov process (SMP) models can incorporate duration dependency. However, left censoring is a common problem in panel studies. In these left-censored surveys, the timing of events occurring before the observation period of the survey are usually unknown and we do not know how long respondents have been in the state in which they were first observed. Current solutions are imperfect. As Guo (1993: 224) notes “When the start time is not known, left truncation remains a very difficult problem unless we are willing to assume a constant hazard rate or to delete all left-truncated subjects”. Assuming constant hazards could be severely biased while discarding observations without known start times leads to substantial loss of information and could be infeasible for survey data where left censored records make up a large proportion of all survey records. Cai et al. (2006) proposed a backward imputation method to simulate the duration elapsed so that they can apply SMP. Nevertheless, this method is computationally heavy, and “the estimates may be dominated by the imputed duration” (Cai et al. 2008:5522) when the follow-up period is short. The problem of left censoring and the computational difficulties and assumptions of current approaches to address it thus add to the substantial gap in understanding and incorporating duration dependence into the calculation of multistate outputs including health life expectancy.

The objectives of this study are twofold. First, we modify and improve the flexibility of the multistate life table with duration dependency (DDMSLT) (Wolf 1988) to make it feasible on left censored survey data. Secondly, this approach enables us to examine the duration effects on HLE by comparing it with the HLE from the typical MSLT. With the comparison, we can uncover whether the bias from failing to incorporate duration dependence is serious in the calculation of HLE.

Methods

Models

The state space of a typical three-state first-order Markov MSLT for estimating HLE is depicted in Figure 1a. There are two transient states (health and unhealthy) and one absorbing state (death). If this model is by single year of age, the health state of the next age would only depend on the health state of the current age. A semi-Markov model (SMP) is very similar to the Markov model except that it assumes that the health state of the next age depends on the state in the current age and the duration in this current state (conceptually shown in Figure 1b). Thus, the probabilities of transitioning from one state to the next in the SMP model are conditional on the length of time spent in the origin state. Duration-specific transition probabilities can be calculated non-parametrically directly from the raw date and they can be modelled parametrically, including by including duration time as a covariate in regression-based models that predict transitions (Cai et al. 2006).

However, to use this model, we need to know the exact duration in each state. As discussed, this is a common problem using many panel datasets where we do not know the duration time that survey respondents have spent in the state in which they are first observed. In other words, we often do not know how long respondents have been in the state they were recorded in at wave 1 of the survey. This is the problem of left censoring. One of the ways we could treat the left censored observations with unknown starting times is to discard all of them (Allison 1984). This method could avoid biases from extra assumptions but at the cost of losing individuals making no transition over the observation period. This could lead to a potential selection bias and the reduced sample characteristic is most likely older than the one in the full data. As a result, the estimates of the older sample could also be biased. The other way to deal with this issue is by the EM algorithm to impute the missing observation before the start of the survey (Cai et al. 2006). Proponents argue that with this approach, SMP models can be estimated based on the full data with the imputed duration. However, the algorithm would need the observed durations to impute the unobserved durations. Thus, the similar selection bias in Allison’s method could be extrapolated to the imputed observations before the censor.

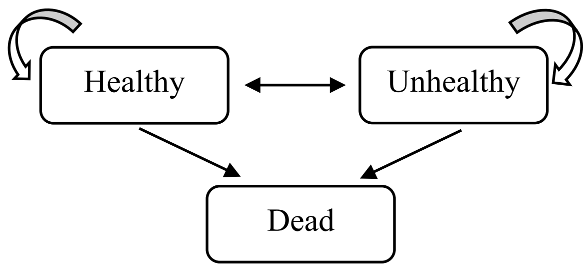

Figure 1a. State space of MSLT with Markov model

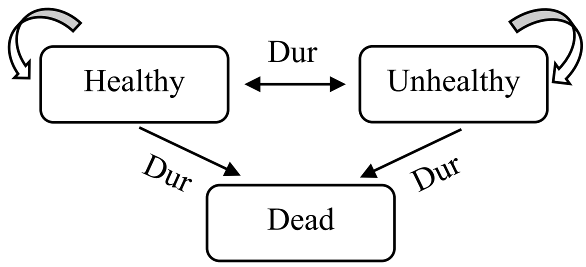

Figure 1b. State space of MSLT with Semi-Markov model
Note: “Dur” represents duration.

The other potential model, duration dependent MSLT (DDMSLT), treats duration as a categorical variable and incorporates duration in each state. Figure 2a shows the state space and the pathways of transition. DDMSLT shares most properties with MSLT, except that the state space is substantially larger because the states are duration-category-specific. Duration increases each age if one stays in the same health status, which is a new state. However, the duration is reset to 0 when one changes to another health status. Also, an individual can die from any state, indicated aggregately as the blue shaded arrows. This model requires a known duration for each health state as well, similar to the SMP model.

Therefore, we propose a small modification, truncated DDMSLT, to allow the model to utilize some of the observations without a known origin. Similar to the Allison method, we drop the observations with unknown origin until a truncated duration. For example, in Figure 2b, this truncation point is set to 3 years of duration. Any observations remaining in the same state for 3 years or more are included in “Dur 3+”. This model can be understood as a piecewise function, when the duration is below a certain bound, it is a semi-Markov process; and when the duration is over the bound, it is a Markov process independent of duration. Because the model incorporates duration into each health state, it is easy to adjust the duration dependency for any specific health state.

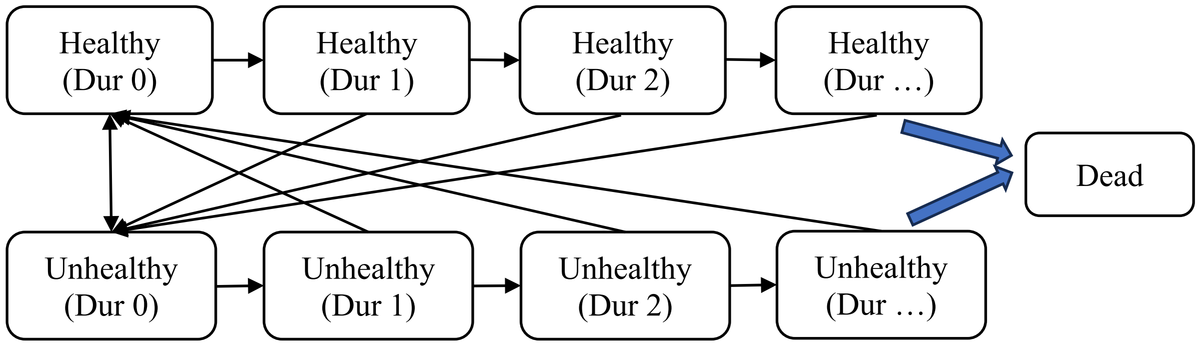

Figure 2a. DDMSLT
Note: “Dur” represents duration. “Dur …” aggregates other states with longer duration. The blue shaded arrows represent transitions to death from all states.

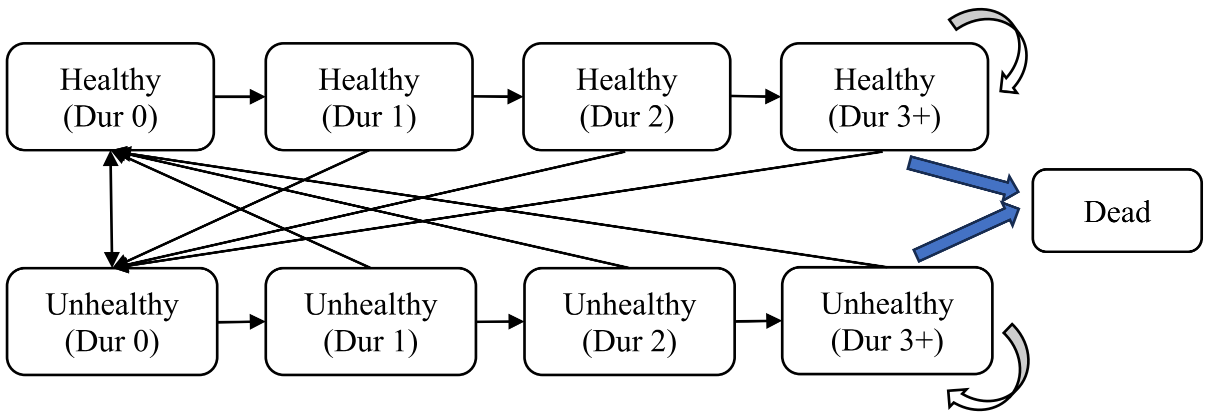

Figure 2b. Truncated DDMSLT (T-DDMSLT)
Note: “Dur” represents duration. “Dur 3+”, as an example, include the states with 3 and above. “3” can be changed to any other truncation of duration. The blue shaded arrows represent transitions to death from all states.

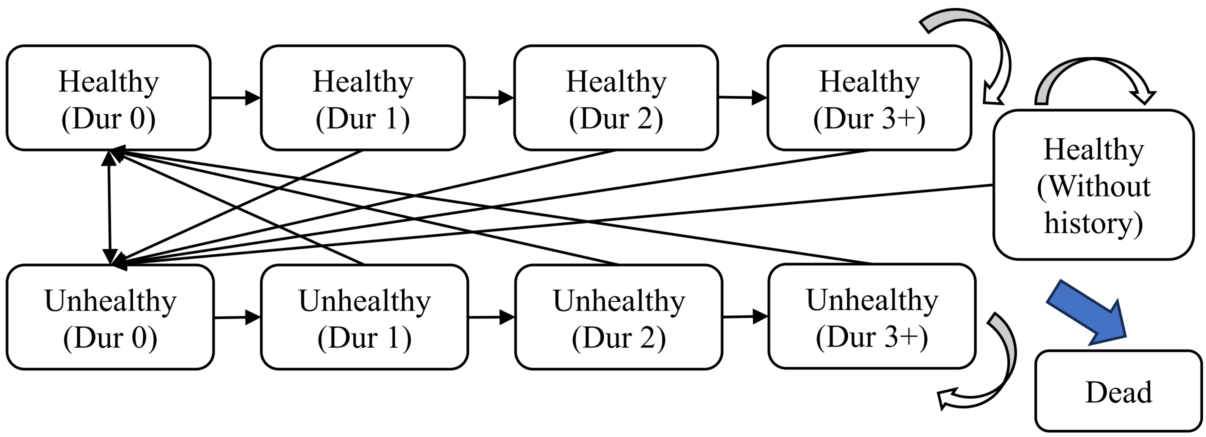

Figure 2c. Truncated DDMSLT with history of unhealthy event (T-DDMSLT-H)
Note: “Dur” represents duration. “Dur 3+”, as an example, include the states with 3 and above. “3” can be changed to any other truncation of duration. The blue shaded arrow represents transitions to death from all states.

A modification to the model utilizing this feature is illustrated in Figure 2c. This model includes a state recording whether an individual with any history of unhealthy events in the past (c.f. Bardenheier et al. 2016). This state is treated as a Markov state where no duration is tracked. As for the other "healthy" state with a history of being unhealthy, the duration can be tracked with the truncation at 3 years of duration as well. However, this duration dependency can also be removed similar to how Bardenheier et al. (2016) constructed their state space. With all these different model designs in Figure 1 and Figure 2, we can estimate the transition probabilities based on empirical data and compare their HLE estimations.

Data

We use data from the US Health and Retirement Survey (HRS), a bi-annual national longitudinal survey. The HRS has a long follow-up period of 15 waves, from 1992 to 2020. However, due to the coverage of cohorts and sampling weight change, we only use the data from wave 5 (2000) to wave 15 (2020). We select a birth cohort, 1936-1945, to estimate the cohort HLE. This cohort is around 60 years old in 2000. To test the model sensitivity to shorter follow-up duration, we also constructed a sub-sample of HRS from wave 5 to wave 9 (2008). In contrast to the cohort HLE, we use this sub-sample to estimate period HLE due to the smaller sample size.

Disability in this study is measure as difficulty in doing the five basic Activities of Daily Living (ADL): bathing, dressing, eating, transferring in/out of bed, and walking across a room. Individuals are classified as unhealthy if they report difficulty on any of the five ADLs (Freedman et al. 2004); otherwise, healthy. HRS also captures death event by linkage with the national registry.

Estimation procedures

Multinomial logistic regressions are used to estimate the age-specific transition probability. Logistic regression is commonly used to estimate discrete time MSLT and SMP models (e.g., Cai et al. 2006; Shen and Payne 2023). These transition probabilities are then inputted to a microsimulation model with a synthetic cohort of 100,000 individuals. For the typical MSLT model, the covariates include age, age-squared, sex and interactions between age and sex. For the T-DDMSLT, we first drop the observations with unknown duration less than the truncation point and group all the states with (known or unknown) duration above the truncation point. Using this technique, we can estimate T-DDMSLT and T-DDMSLT-H with different truncation points. In the results, we only show the truncation with 3 and 5 years. However, to ensure the samples used in different models are comparable, we constrain the sample to the one with truncation of 5 years. We also estimate another MSLT model with the constrained sample. For the SMP model, we remove all the observations with unknown origin and fitting the result of the sample. Therefore, there are three subsets of data used in the estimation: 1) full data; 2) data dropping unknown duration of 5 years; 3) data without unknown origins. Bootstrap resampling from the original dataset is used to generate confidence intervals (CIs). Based on these 500 bootstrap samples, we re-estimate and simulate. The final point estimates reported in the results are from the entire dataset, and the 95% CI is taken as the central 95% of the 500 bootstrap resamples times.

Results

The results are mainly based on the cohort perspective with 20 years of follow-up. First, we look at the males' transition probabilities in different models in Figures 3 and 4. Figure 3 shows age-specific transition probability from 65 to 80 by current health state, with healthy on the left panel and unhealthy on the right. The colour of the lines represents the next state. For example, the probability of remaining healthy (red line on the left panel) is about 0.95 at age 65. The dominant transitions are typically remaining in the same state be it healthy or unhealthy.

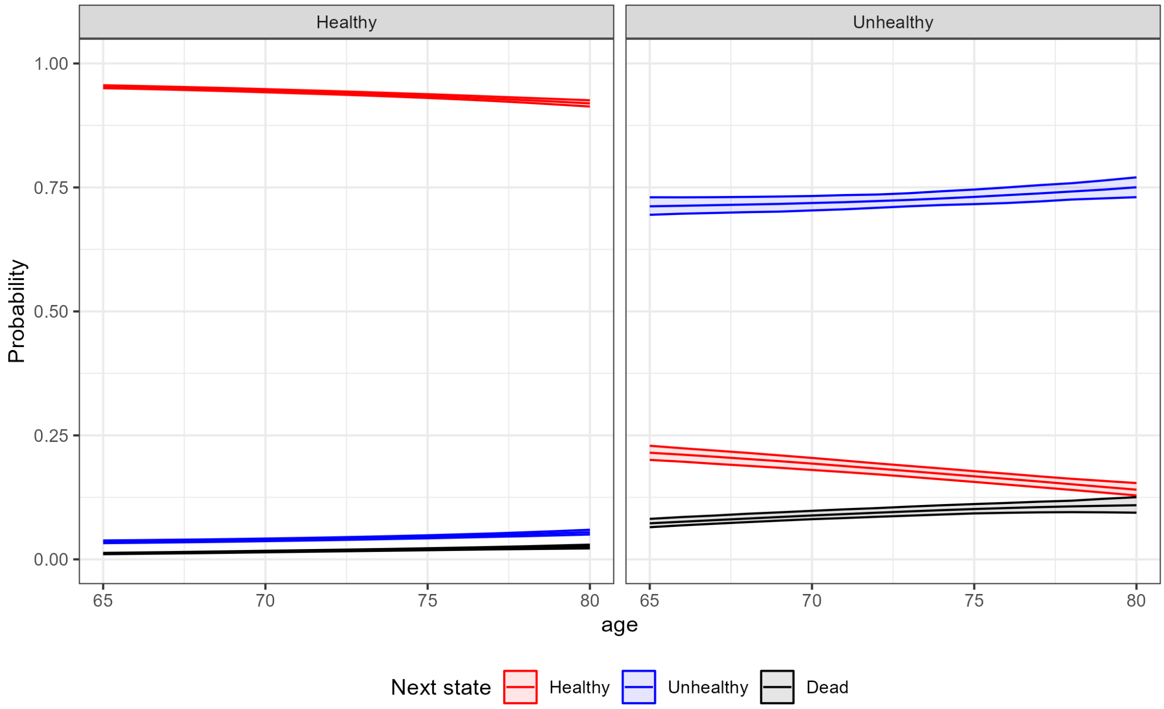

Figure 3. Age-specific transition probabilities for male cohort 1936-1945 with MSLT (Full)
Note: 95% CIs are in the shaded area.

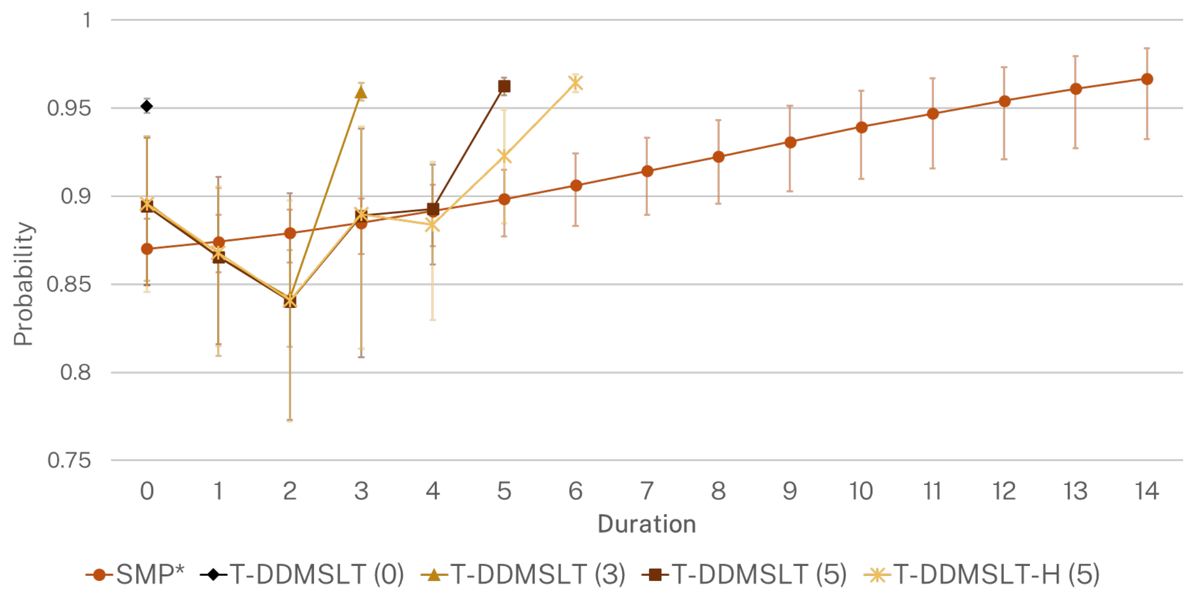

Figure 4a. Probability staying healthy at age 65 for male cohort 1936-1945
Note: Duration at 6 for the model T-DDMSLT-H (5) refers to the special state: healthy without history of unhealthy events. 95% CIs are in the error bars. SMP model is estimated using the subsample without any unknow duration.

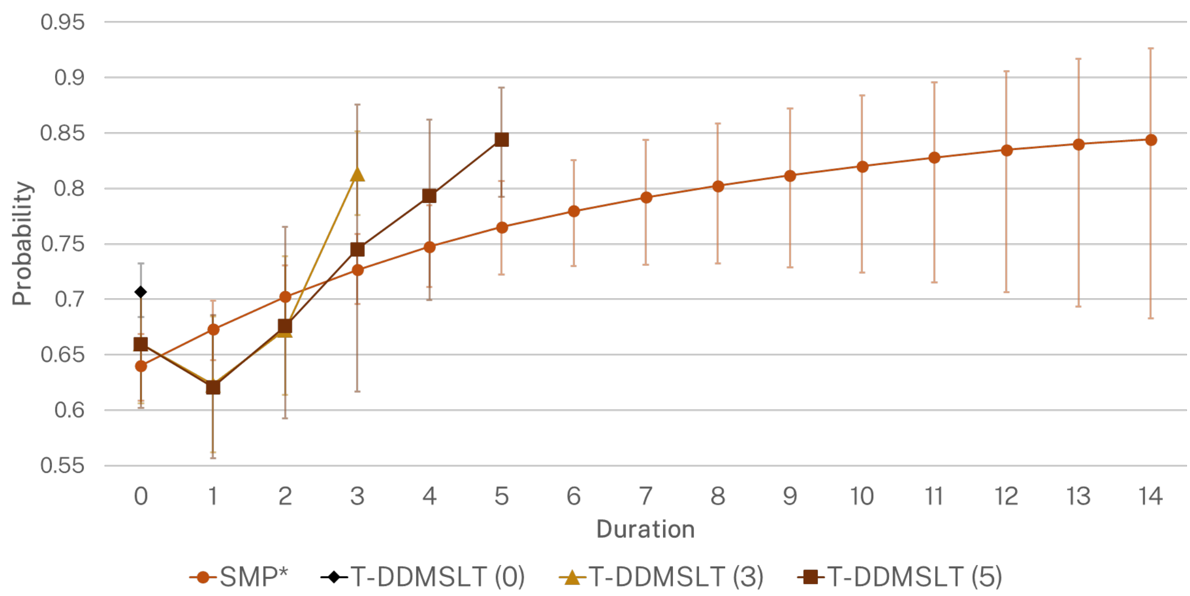

Figure 4b. Probability staying unhealthy at age 65 for male cohort 1936-1945
Note: 95% CIs are in the error bars. SMP model is estimated using the subsample without any unknow duration.

Since the age patterns are rather smooth, we focus on the duration in each age rather than the age pattern in Figure 4. Figure 4 shows the transition probability of remaining healthy (in panel a) and remaining unhealthy (in panel b) for male at 65 in cohort 1936-1945. Noted that the T-DDMSLT (0) is the same as model. However, T-DDMSLT (0) is different from the MSLT in Figure 3, because it is estimated with data dropping unknown duration of 5 years. Additionally, the SMP model used another subsample without any unknown duration. Because the duration is estimated as a continuous function in SMP, it is smoothed across all durations.

The transition probabilities in the SMP and T-DDMSLT models are significantly lower for shorter durations and increase as duration increases in comparison to the MSLT. The probability of remaining healthy, however, increases at a slower rate over time than the probability of staying unhealthy. In both panels, the transition probabilities in the T-DDMSLT model gradually decline over the first few durations before gradually increasing over greater durations. Figure 4a shows that the likelihood of being healthy increases after two years, while Figure 4b shows that the likelihood of remaining unhealthy increases after one year. In particular, the transition probability at the truncation duration is significantly higher than the MSLT estimate. In other words, the stickiness is higher for long-term state, but the fluidity is also higher for the short-term state compared to the Markov MSLT model. For the model recording history of unhealthy event (T-DDMSLT-H (5)), the duration at 6 in Figure 4a represents the special state: healthy without history of unhealthy events. After the separation of individuals without history, the probability of staying healthy for duration 5 or above is much lower than the T-DDMSLT (5) model. It indicates that the people with history are probability frailer and more likely to transition out of healthy state.

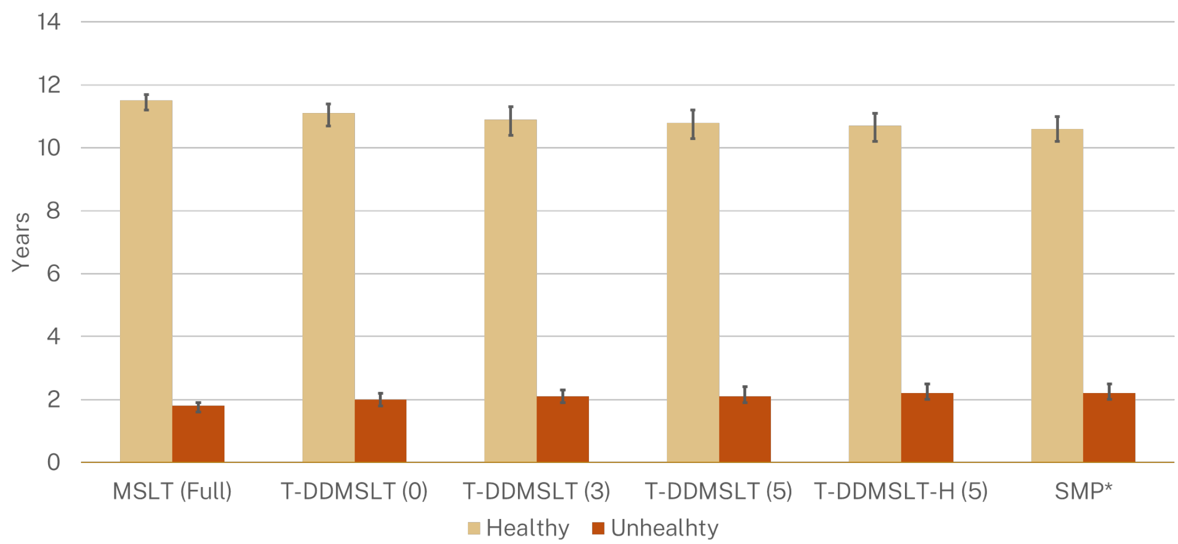

Figure 5. Partial healthy life expectancy of male cohort, age 65-80

The partial cohort life expectancy from 65 to 80 can be calculated using the microsimulation with the probability from each model. The first MSLT model makes use of the entire set of data, and while its unhealthy life expectancy (ULE) is much lower than the other models', its HLE is noticeably greater than that of those other models. However, all of the T-DDMSLT exhibit comparable HLE and ULE once the sample was restricted to a truncation of 5 years. Additionally, even using an even more constrained sample than the T-DDMSLT, the SMP model's estimation is not statistically different from that of the T-DDMSLT (0) (i.e., the MSLT with a 5-year truncation).

We can calculate the duration remaining life expectancy at age 65 based on the shorter follow-up subsample and similar approaches. After taking into account the uncertainty, the HLEs are quite comparable across all models. In contrast to the T-DDMSLT and SMP models, the MSLT model using the complete dataset would generate a substantially lower ULE. This result is strikingly similar to that of Cai et al. (2006), who suggest that their EM-SMP model results in a longer period of disability than the traditional MSLT model. It is worth noting that HLE is slightly lower than the other T-DDMSLT if the history is recorded, though it is not significant.

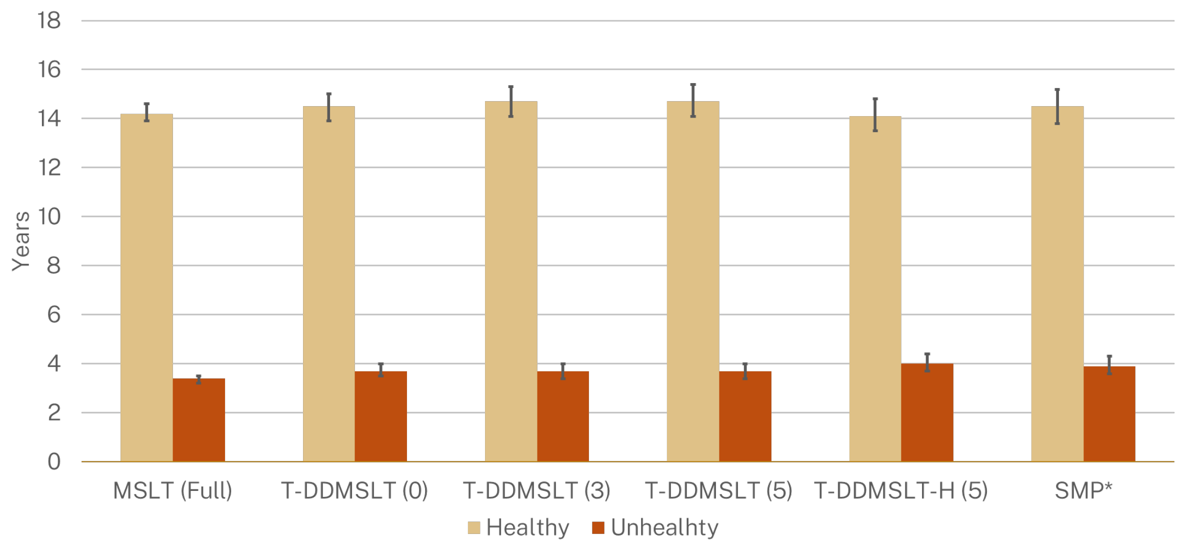

Figure 6. Remaining healthy life expectancy at 65 for male in 2000

Discussion and Conclusion

There are observable differences in multistate transition probabilities when duration dependency is taken into account. To be more precise, the probability of remaining in the same state is lower compared to the Markov MSLT model, but it rises and becomes higher as the duration remaining in the same state increases. These trends have also been seen in other studies, including those by Cai et al. (2006, 2008). Yet, the discrepancy in transition probability may not translate to differences in the healthy life expectancy, which appears to contradict with Cai et al. (2006).

The important point to note is that MSLT could estimate a very similar healthy life expectancy to the one with duration dependency, be it T-DDMSLT or SMP if they are all based on the sample without some (or all) of the left-censored observations. The difference between MSLT and models with duration dependency only appeared in this study when the sample for the estimation was different. As discussed in the foregoing passage, the sample characteristics are likely to be incomparable. One of the most prominent and intuitive distinctions would be age because duration can only be observed after one transition from a different state.

Patterns in duration can explain why transition probability differ between models yet healthy life expectancies are comparable. While stickiness to the same state may be lowered in shorter-duration states, stickiness is greater in longer-duration states. MSLT averages out the impacts of duration dependency since it uses data with all durations. As a result, after controlling the data, the estimates with and without duration dependency are comparable. However, if the focus is on life course trajectories rather than life expectancy, the simulation with duration may produce different findings from the simulation with the Markov assumption. This might be a future study topic for more investigation.

In conclusion, we indeed, also find duration dependency in the different models tested. However, in estimating healthy life expectancy in this study, any bias induced by not considering duration dependence is not so serious when the sample for the estimation is the same. As discussed by Guo (1993), the left-censored design is practically an intractable issue. There is no model being a gold standard without its specific assumptions and limitations. Given that the standard MSLT can produce comparable estimates to the different models with duration and also utilizing the most observed data, for the simplicity of the model, the Markov model is a sound approach to estimating healthy life expectancy. This finding though is specific to the context of this study and future research may look to explore conditions under which duration dependency does have a meaningful impact on life expectancy calculation.

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