EPC 2024 Submission

Life Expectancy Decomposition:

The contribution of changes in numerators and denominators.

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Abstract:

Demographers use ratios, proportions, and rates as the main input of their research, all of which are calculated as counts of numerators divided by denominators. In the case of age-specific death rates, the denominator is the person-years lived in the period of study, and the numerator is the deaths that occur during the period. Period life expectancy summarises those rates into one measure and its changes convey messages of changing mortality between periods, as well as showing progression/deterioration of longevity. In the past, methods have been developed to measure the contributions of different age groups, different subnational groups, and different causes of these changes in period life expectancies. Here we present an alternative perspective that examines the contributions from the two fundamental components to the dynamics of life expectancy changes: the population growth of the person-years in the denominator, and the relative changes in number of deaths in the numerators. Our results show that population growth at older ages increases life expectancy through the denominator component. At the same time, life expectancy changes varied across populations due to the different age patterns of the numerator component.

Introduction

Demographers use ratios, proportions, and rates as the main input of their research, all of which are calculated as counts of numerators divided by denominators. In the case of age-specific death rates, the denominator is the person-years (or mid-year population) lived in the period of study, and the numerator is the deaths that occur during the period. Understanding age-specific mortality, particularly the interplay between the numerator and denominator, is essential for calculating various demographic indicators, including period life expectancy in mortality research.

Period life expectancy summarises those age-specific mortality rates into one measure and its changes convey messages of changing mortality between periods, as well as showing the trends in population longevity. Over time, demographers have developed demographic decomposition methods to assess the contributions to changes in period life expectancies. These methods can disentangle the influences of various age groups (Arriaga, 1984; Pollard, 1988), different subnational groups (Shkolnikov et al., 2006; Torres et al., 2019), and different causes of death (Vaupel & Canudas-Romo, 2003; Beltran-Sanchez, Preston & Canudas-Romo 2008).

Nevertheless, there's been limited focus on separating the contributions to changes in life expectancy regarding the fundamental structure of the input values, namely the numerators and denominators in the age-specific death rates. Thus, we present an alternative perspective that examines the contributions from the two fundamental components relating the input values of age-specific mortality to the dynamics of life expectancy changes: the population growth of the denominator, and the changes in the numerator, representing the observed number of deaths.

Data and Method

We used the data from HMD (2023) to illustrate the effect of the numerator and the denominator on life expectancy changes among four female populations: Australia, Denmark, Japan, and the USA during the period 2009-2019.

Canudas-Romo et al. (2022) proposed decomposing changes in age-specific death rates into the numerator and denominator contributions. The core idea of the approach is to study the changes in growth of the exposed population (the denominator) and the relative change in the absolute number of deaths (the numerator) between the two periods. Let a dot on top of a variable denote the derivatives with respect to time (Vaupel & Canudas-Romo, 2003). The changes in life expectancy at birth at time *t*, or $e_0(0,t)$, can be expressed as

$$\dot{e}_0(0,t) = \int_0^{\omega} f(x,t)e_0(x,t)[r(x,t) - r_D(x,t)]\ dx, \qquad (1)$$

where $r_D(x,t)$ corresponds to the relative change in the observed number of deaths, we refer to this as the growth rate of observed deaths $D(x,t)$ at age *x* and time *t*, $r_D(x,t) = \frac{\dot{D}(x,t)}{D(x,t)}$; the $r(x,t)$ is the age-specific growth rates, or $r(x,t) = \frac{\dot{P}(x,t)}{P(x,t)}$, with population denoted as $P(x,t)$; finally $f(x,t)$ and $e_0(x,t)$ correspond to the life table distribution of deaths and remaining life expectancy at age *x* and time *t*. The notation $f(x,t)e_0(x,t)$ can be seen as weights that reflect the age-specific impact of survival improvements on life expectancy. We can then disentangle this equation into a numerator component of growth rates of deaths, $-\int_0^{\omega} f(x,t)e(x,t)r_D(x,t)$, and a denominator component of population growth, $\int_0^{\omega} f(x,t)e(x,t)r(x,t)$. The two components can be further disaggregated by age.

Preliminary Results

Table 1 presents the life expectancy and its change from 2009 to 2019 for the four selected countries. All these populations saw increases in their life expectancy, although the reasons for such rise varied from country to country.

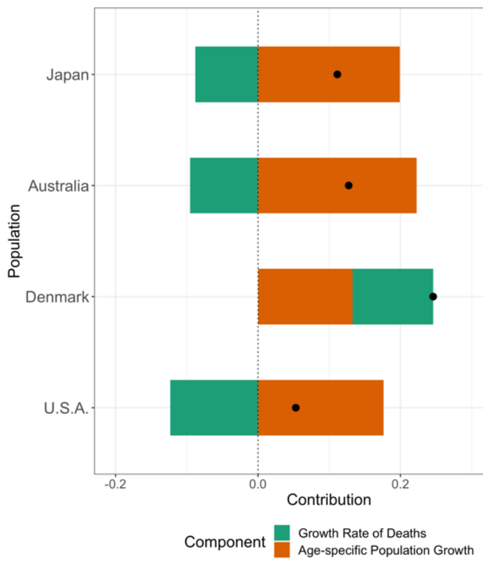
Table 1. Life expectancies for 2009 and 2019, and their annualized changes during the period for selected female populations.

Population	$e_0(2009)$	$e_0(2019)$	$e_0(2019)$- $e_0(2009)$
Japan	86.3	87.4	0.12
Australia	84.0	85.3	0.13
Denmark	81.0	83.4	0.25
U.S.A.	80.9	81.5	0.05

Source: based on the HMD (2023), ordered from highest to lowest life expectancy.

Figure 1 presents the breakdown of the annualized changes in life expectancy into the numerator and the denominator components for the four selected populations. In populations where life expectancy was relatively high, like Japan and Australia, the increase in life expectancy was due to a growing population (the denominator component) and fewer deaths (the numerator component). The USA shows similar contributions, even though its life expectancy was lower. For Denmark, the denominator and numerator components both contributed to an increase in life expectancy.

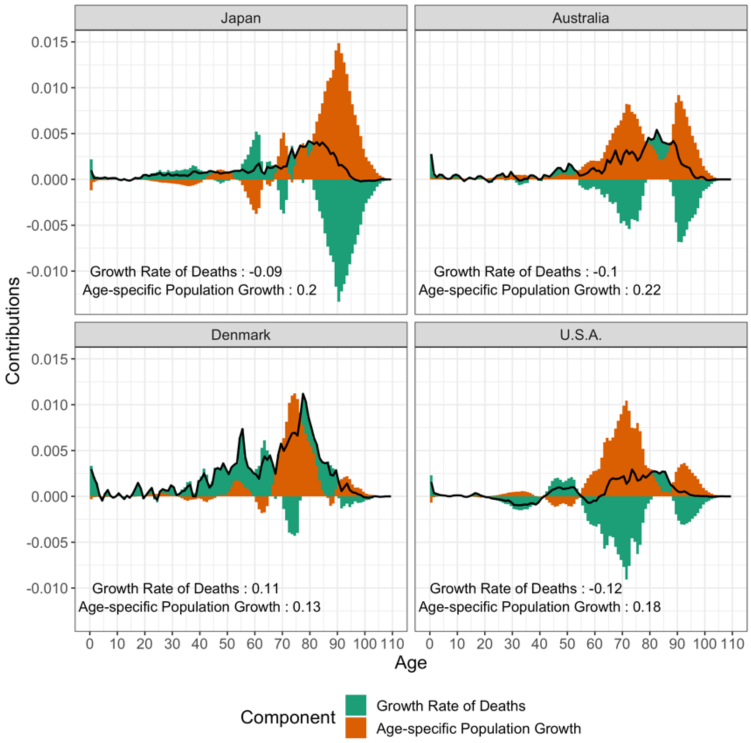
Figure 1. Decomposition of changes in life expectancies for selected female populations between 2009 and 2019 into growth rates of deaths and population components.



Source: author's calculations based on the HMD (2023).
Notes: The black dot represents the annualized changes in life expectancy during the period 2009-2019 for each population (as in Table 1). Ordered as in Table 1.

Figure 2 shows the age-contributions for both factors for each of these populations. Japan's life expectancy rose mainly because the elderly population (ages 80-90) grew, increasing the denominator. This had a bigger impact than the growth in deaths among the very old (ages 90+). Australia displayed a similar age pattern, but it started at younger ages (ages 60-75 and ages 90). For Denmark, a similar increase in population growth component at older ages (also observed in Japan and Australia), and a decrease in the number of deaths at younger ages, both contributed to an increase in life expectancy. In the US, more deaths among people aged 20-40 and 55-80 (see black line of total age-contribution) slowed down the increase in life expectancy compared to the other populations.

Figure 2. Age-specific decomposition of changes in life expectancies for selected female populations between 2009 and 2019 into growth rates of deaths and population components.



Source: author's calculations based on the HMD (2023).

Notes: Total contributions, as seen in Figure 1, are shown as subscripts within the figure panels. Black line corresponds to the total age-contribution of the change in life expectancy.

Next Steps

We plan to extend these relationships to other HMD populations as well as study the changes in cohort life expectancy.

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