A Three-Component Model for Adult Mortality

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Abstract

We introduce a model that disaggregates adult mortality into three components: senescent, background, and behavioral. We assume the latter results from the interaction of one's inclination to act risky and the associated damage. The suggested model is quite general and can incorporate unobserved heterogeneity, as well as dependencies among the components. We study several model specifications and compare their goodness-of-fit on data from the Human Mortality Database. We also estimate the age-specific share of deaths pertaining to each component.

Introduction

Disentangling senescent from premature deaths in a mortality regime has at least two advantages: first, researchers can study the aging process by focusing on the senescent mortality component only, and second, they can estimate the share of non-senescent deaths in the population by age or some other stratifying variable to measure the impact of premature deaths on overall mortality. In this study, we distinguish between two non-senescent hazards: a general background hazard and a behavioral hazard reflecting mortality due to acting risky.

The Model

Suppose the force of mortality $\bar{\mu}(x)$ for a given population at age x is given by

$$\mu(x) = \mu_S(x) + \mu_{GB}(x) + \mu_{RB}(x)\,, \tag{1}$$

where $\mu_S(x)$ denotes the risk of dying from senescent causes, $\mu_{GB}(x)$ stands for the general background mortality not related to the aging process, and $\mu_{GB}(x)$ reflects non-senescent mortality resulting from one's risky behavior. While this model specification is quite general, we can assume that $\mu_S(x)$ is a gamma-Gompertz hazard, i.e.,

$$\mu_S(x) = \frac{ae^{bx}}{1 + \frac{a\gamma}{b}\left(e^{bx} - 1\right)}\,, \tag{2}$$

where a is the level of mortality at the starting age of analysis, b is the rate of aging, and γ is the squared-coefficient of variation of the frailty distribution (Vaupel et al., 1979; Vaupel and Missov, 2014). We can also assume that the constant Makeham term c (Makeham, 1860) captures the background risk of dying, i.e.,

$$\mu_{GB}(x) = c\,. \tag{3}$$

For $\mu_{GB}(x)$, we assume that it is of the form $\mu_{GB}(x) = f(x)\,g(x)$, where $f(x)$ is a function reflecting one's inclination to act risky, while $g(x)$ is the damage resulting from an act of risky behavior. Intuitively, we assume that $f(x)$ is declining with age while $g(x)$ increases. Suppose $g(x) = \mu_S(x)$, i.e., the damage from acting risky is directly linked to the aging process. For $f(x)$, we assume that $f(x) = \eta\, S(x)$, where η is a scaling parameter and $S(x)$ is a survival function:

$$\mu_{RB}(x) = \eta\, \mu_S(x)\, S(x)\,. \tag{4}$$

Model Estimation and Preliminary Results

We use raw death counts $D(x)$ and exposures $E(x)$ from HMD (2023), assuming that $D(x) \sim$Poisson$(\mu(x)\, E(x))$ (Brillinger, 1986). To estimate the model (1), we apply a Bayesian procedure with gamma priors for the parameters (Patricio and Missov, 2023).

The first step is to choose an appropriate survival function for the behavioral component. We assume an exponential, a gamma, a Rayleigh, and a log-normal distribution for $S(x)$, compare the goodness-of-fit of (1) under each $S(x)$ specification, and choose the one providing the best fit. To determine the latter, we consider seven measures: MSE (Mean Square Error Loss), RMSE (Root Mean Square Error Loss), RMSLE (Root Mean Squared Logarithmic Error Loss), RAE (Relative Absolute Error Loss), MAE (Mean Absolute Error Loss), MAPE (Mean Absolute Percentage Error Loss), MedianAPE (Median Absolute Percentage Error Loss). For French females aged 30 and above in 1970, the log-normal survival function provides the best fit according to five of these seven measures, and therefore we choose it for modeling $S(x)$ in (4). Figure 1 shows the observed and estimated by (1) overall mortality, as well as its decomposition into its senescent, background, and behavioral components.

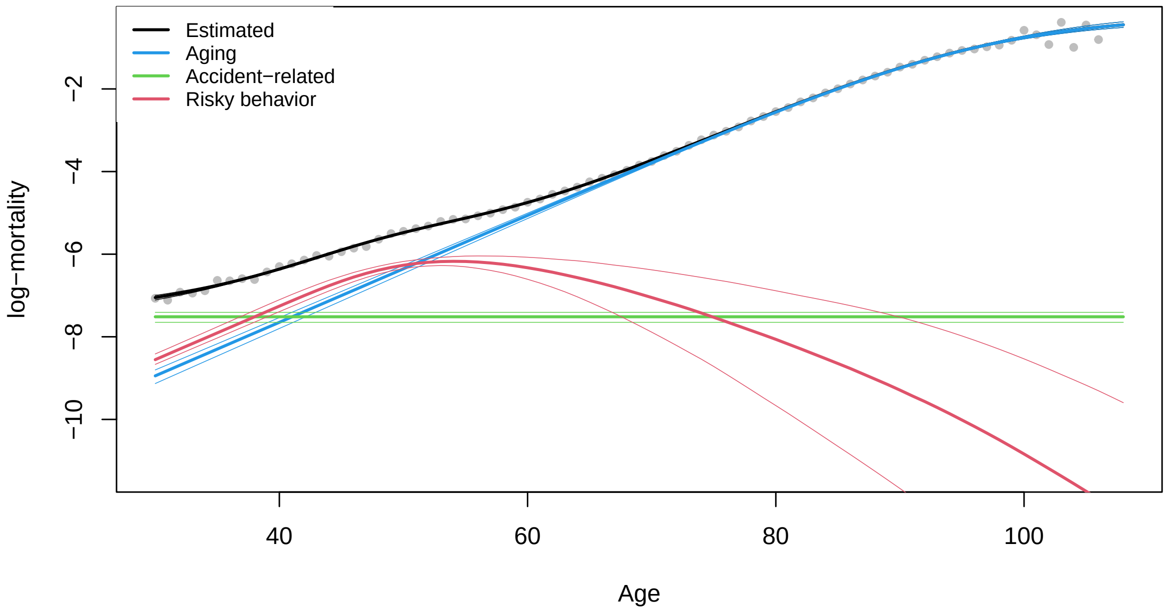

Figure 1: Mortality of French females after age 30 in 1970. Overall mortality (observed: circles, fitted: black line) is decomposed into three estimated components: senescent (blue), general background (green), and behavioral (red).

Figure 2 shows the share of deaths by each of the three components for French females in 1970. The non-senescent components dominate overall mortality until about age 60, and it is quite remarkable that the behavioral component is leading after age 40. From age 80 onwards model (1) estimates a negligible share of non-senescent deaths.

Discussion and preliminary conclusions

Wisser (2015) suggests a simplified version of model (1), but fails to estimate it as fitting by a standard maximum likelihood is not statistically feasible. Here, we extend Wisser's model by suggesting a more

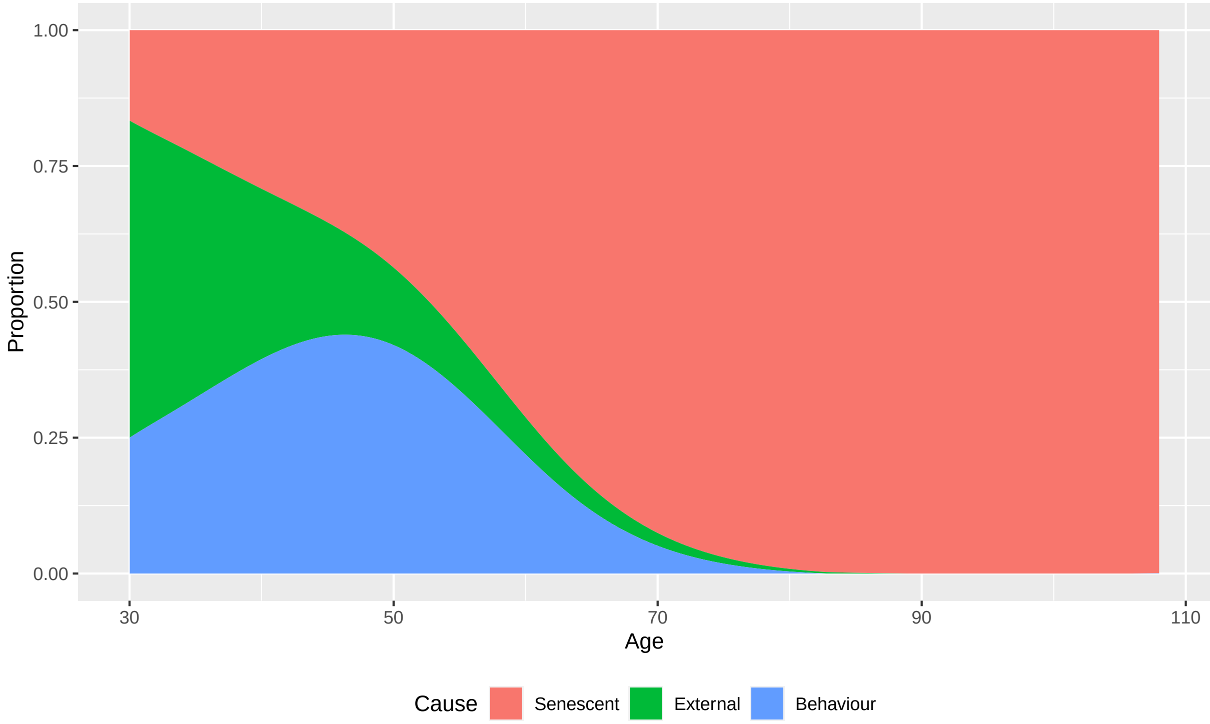

Figure 2: Share of deaths by senescent, general background and behavioral components of mortality for French females after age 30 in 1970.

complex form for $\mu_{RB}(x)$ and adding a Makeham term. Moreover, we propose a Bayesian procedure for its estimation. As further steps we plan to (i) fit the model to the major causes of death (COD) to identify which COD are "senescent", (ii) use the estimated senescent mortality to characterize the aging process.

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