A Sextic Representation of the Mortlity Curve

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There is a pattern of age specific mortality which, under normal circumstances, is common to all life tables: the rapid decline in infancy and childhood, the minimum attained at about age 10, the increase to a possible plateau in early adulthood, and the gradual rise in the level of mortality as senescence is approached. Details will differ, however. As mortality declines, not only does the mortality curve move down the y axis, but the relative strengths of the different components of the curve change. When mortality is high, mortality rates are particularly high at young ages, as half or more of those born may die before age five. As mortality declines, there is a growing concentration of deaths at the upper end of the age scale with a consequent pushing back of the modal age at death (Fries, 1983; Wilmoth & Horiuchi, 1999). However, not all populations follow the same path. In previous work (Anson, 1988, 1991, 1992, 1993a, 1993b) I have suggested that human life tables can be located in a two-dimensional space, and that just two parameters are required in order to identify a life table uniquely and to distinguish it from all others: one parameter to describe the general level of mortality in the population, and one to describe the specific shape (morphology) of the curve, net of the level of mortality. In the present paper, I wish to look more closely at the parameters describing the mortality curve and at the relationships between them. Using this information, I shall consider how we may map the changing shape of the mortality curve as mortality declines, and how we may distinguish between the paths different populations take as their mortality declines.

Figure 1: Mortality Curves, Sweden, Males, 1800 — 2000

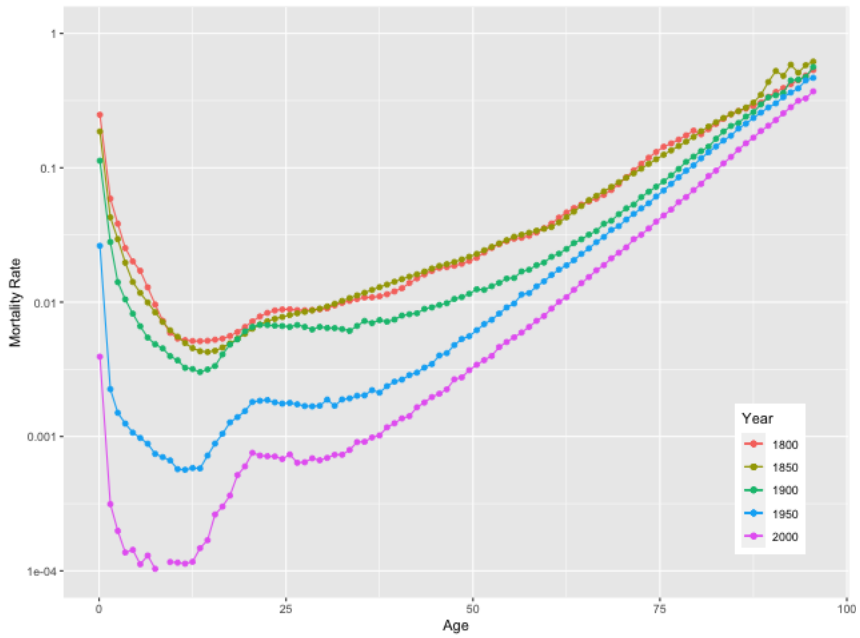


Figure 1 presents five (male) mortality curves for Sweden, over a span of 200 years, from around 1800 to around 2000 (each curve is composed of mortality for the decade around the focal year¸thus 1795-1804, etc.). The curves for 1800 and 1850 are almost identical¸but from the middle of the 19th century mortality declines. This decline is more noticeable at younger years, infant mortality in particular drops by a factor of almost 100; the age of minimum mortality declines from around 15 to less than 10 but the local maximum in late teens, at around age 20, remains fairly constant; there is a small relative decline in old age mortality in the latter half of the 19th century but it then stays fairly constant till the late 20th century. At very old ages, 100 and above, when the number of persons still alive is very small, and at very low levels of mortality, especially in recent years, when rates are exceedingly low, there is a lot of variation in the recorded rates.

We may take two approaches to modelling the mortality curve: a global approach and piecemeal approach. In the global approach we fit one function to the whole of the life span. The fit will not be perfect, but fitting the curve will be fairly straightforward and it will, we trust, capture the main characteristics of the curve and enable us to see how it evolves over time. The alternative, piecemeal approach, will fit a series of curves, each of which will be salient at a certain period of the life span and negligeable at all other ages. The fit will probably be better than in the global approach, but the actual fitting of the curve is liable to be more problematic. We shall focus here on the global approach to see how its parameters can enable us to understand the evolution of the mortality curve over time and how it is related to the social conditions in different countries and regions.

The Global Approach: A quartic equation

Models fitting the mortality curve have suggested the need for 6 or more parameters thus suggesting we need (at least) a sextic (degree 6) curve for its representation. A general form for the sextic curve might be

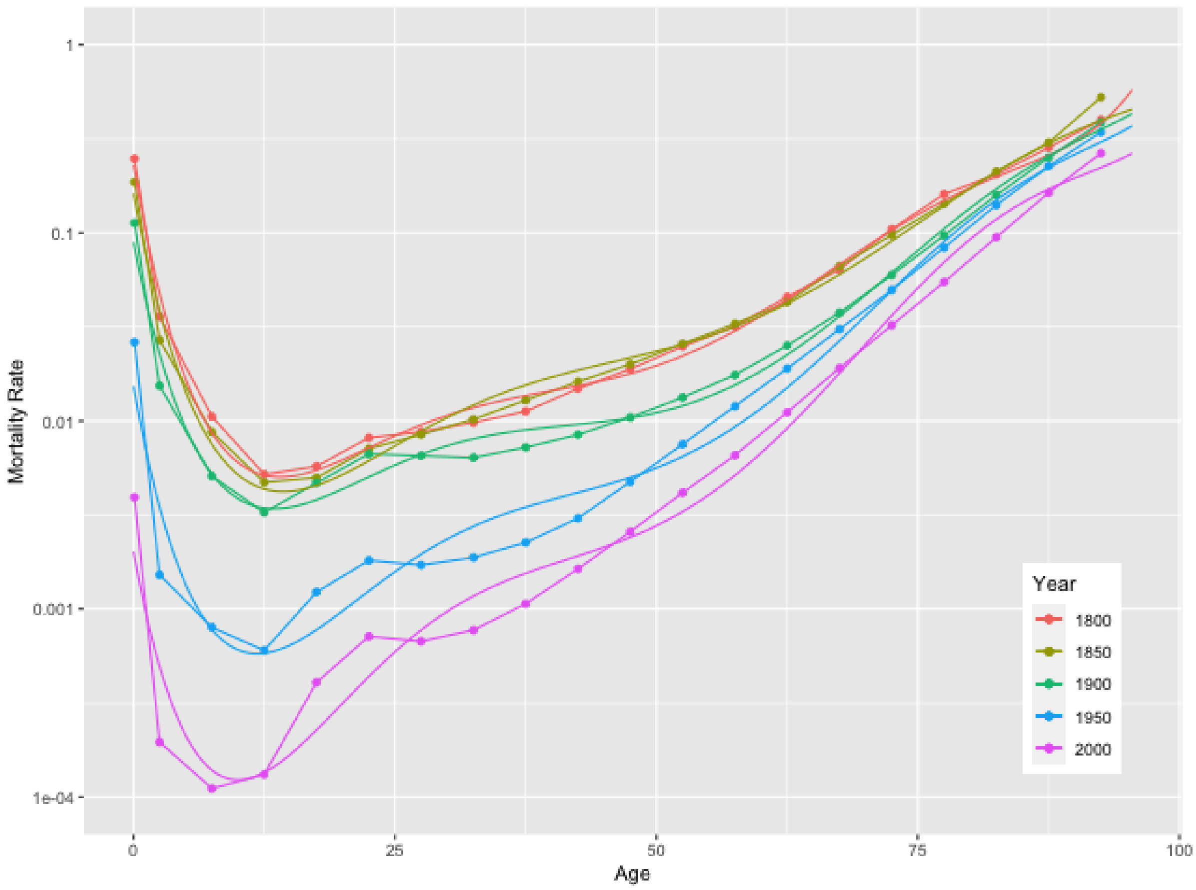
$$y = X^6 - \beta X^4 - \phi X^2 + \tau X \tag{1}$$

where y represents the log of the mortality rate, β represents the *breadth*, or width, of the curve, how stretched out it is between it lower and upper arms; ϕ represents its *flatness*, the force pushing up below its central point, and τ represents the *tilt* of the curve, the height of the right hand arm relative to that of the left hand arm. In this equation, X is generic. In order to locate and scale it with respect to the age and mortality axes, we need to *centre* it around a central point, ξ, and *scale* the axis with a parameter σ. The actual *location* of the curve at its central point will be given by λ. The curve to be evaluated will thus be

$$y = [\sigma(x-\xi)]^6 - \beta[\sigma(x-\xi)]^4 - \phi[\sigma(x-\xi)]^2 + \tau\sigma(x-\xi) + \lambda \tag{2}$$

where $x$is the age axis, centred and scaled by ξ and $\sigma$and y is the log of the mortality rate, evaluated at ages 0.1, 1.5, 2.5, . . . 95.5. We end the computation at this age in view of the instability of observed rates above this age. By scaling and centering the x-axis, ϕ and $\tau$are characteristics of the curve which are independent of the specific level of mortality, which is encapsulated in the other three parameters. Figure 2 fits these curves over the empirical curves of Figure 1. Although these curves cannot reprodue the detail, they do reproduce the major functional form of the curves.

Figure 2: Mortality Curves, Sweden, Males, 1800 — 2000with Sexic fits



We computed parameters for all life tables in HMD, using the abbreviated life tables at 5-year intervals, in total 1752 life tables (876 male and 876 female) from 45 countries. We now consider briefly the relations between these 6 parameters

1. The scaling and location parameters¸σ, and ξ : These parameters were collinear (r = -0.909) formed just one factor which correlated well with e_0¸particularly since the late 1940's, and was collinear with τ(tilt). Over time the scale (σ)paramtere has been declining, and the centre (ξ) parameter has been increasing¸particularly since the turn of the 20th century.

2. The location(λ) parameter has a bifurcated history. During the 19th and 20th centuries it was rising steadily, in line with the rise in life expectancy. Since the turn of the 21st century, however, a new pattern has developed, which does not appear to bear a direct relationship with the level of mortality.

3. The tilt parameter (τ), which we treat separately as it is a basic structural parameter of the curve, has been increasing steadily over time, in line with the increase in life expectancy. As may be expected, as mortality declines, but maximum life span remains essentially constant, deaths become concentrated in the upper end of the mortality curve and the curve becomes more tilted with a lower left hand arm and a relatively greater right hand arm. As with all the measures of the level of mortality, there is a high correlation (r = 0.9) between the tilt of the male and female mortality curves.

4. The last two parameters¸β and ϕ, the *breadth* and the *flatness* of the curve, control the age range over which mortality is low,before it begins to rise into old age. These two parameters and closely correlated (r = 0.907) and may be combined to form what we shall call the Shape parameter. This is only lightly related to the other parameters and, in particular, shows little variation over time, though in the past century its variablility has increased considerably. It also shows variablity between countries. Although there is considerable overlap, female mortality curves tend to be slighly flatter and broader than the corresponding male curves. Unlike the Level parameters¸the correlation between the Shape of the male and female mortality curves is much lower (r = 0.6)˙

Conclusion

The sextic curve provides a close fit to a broad variety of mortality curves. By defining the curve in terms of three structural parameters (the breadth,the flatness and the tilt) and three scaling and location parameters (scale, centre and location)we obtain a transparent description of the mortality curve¸which enables us to describe the changes that have taken place over time, in relation to the general level of mortality in the population and the social conditions in the different countries over time. The final paper will attempt to elucidate some of these relationships and, in particular, identify the social conditions giving rise to differently shaped mortality curves.